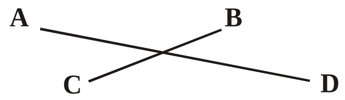
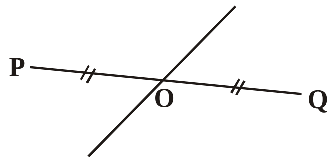
Background knowledge – Revision – Grade 8 Geometry

N1 Lines and angles:

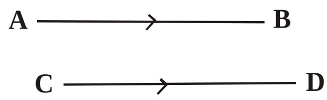
(1) Secants: Two lines intersecting.
$\therefore$ AD intersects BC.



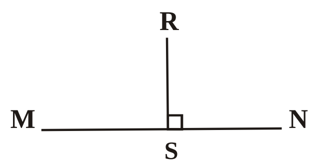
(2) Bisector: One line intersects another exactly in the middle.
$\therefore$ PO = OQ.



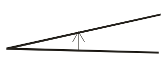
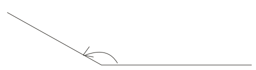
(3) Parallel lines: Two or more lines that are always the same distance apart and will never cross each other.
$\therefore$ AB // CD.



(4) Perpendicular lines: A line is perpendicular to another line if it forms a 90° angle with the other line.
$\therefore$ RS $\perp$ MN.

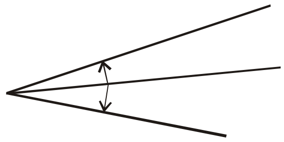


(5) Types of angles:

Name of angle:	**Example:**	**Size of angle:**
Acute angle		Greater than 0° but smaller than 90°.
Right angle		Equal to 90°.
Obtuse angle		Greater than 90° but smaller than 180°.
Straight angle		Equal to 180°.
Reflex angle		Greater than 180° but smaller than 360°.
Revolution		Equal to 360°.

(6) Adjacent angles:

Two angles with a common vertex and a common arm, and the two angles lie on either side of the common arm.

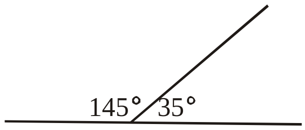


(7) Complementary angles: Angles with a sum of 90°.

(8) Supplementary angles: Angles with a sum of 180°.

(9) Angles on a straight line:

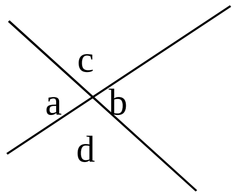
The sum of the adjacent angles on a straight line is 180°.



(10) Vertically opposite angles:

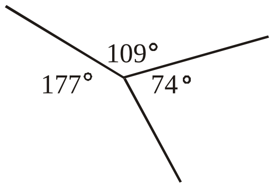
If two lines intersect each other, the vertically opposite angles are equal in size.

$\therefore \quad \hat{a} = \hat{b} \quad and \quad \hat{c} = \hat{d}$



(11) Angles around a point (revolution):

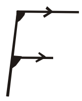
The sum of the angles around a point is 360°.



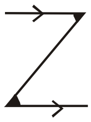
(12) Parallel lines:

If two lines are parallel, then the following will apply:

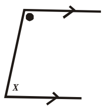
(1) Corresponding angles:
E.g. a = e; b = f;
c = g and d = h.

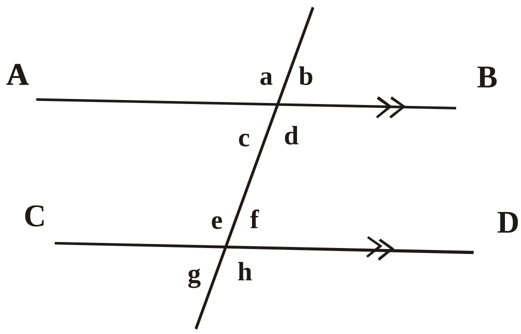


(2) Alternate angles:
E.g. e = d; c = f;
a = h and b = g.



(3) Co-interior angles:
E.g. c + e = 180° and
d + f = 180° .

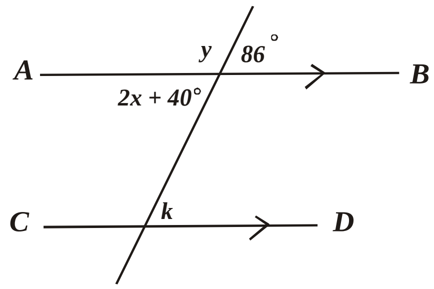




(13) To prove lines are parallel, one of the following has to apply:

(a) A pair of corresponding angles must be equal or
(b) A pair of alternate angles must be equal or
(c) A pair of co-interior angles must be equal to 180°.

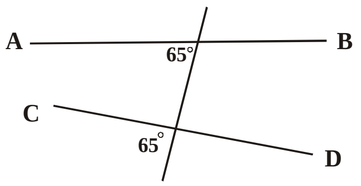
E.g. 1 (a) Calculate the values of the variables; provide reasons:



* $y = 180° - 86°$ [$\angle^s$ on straight line]
  $\underline{y = 94°}$
* $2x + 40 = 86°$ [vert. opp. $\angle^s$]
  $2x = 86° - 40°$
  $2x = 46°$
  $\underline{x = 23°}$
* $\underline{k = 86°}$ [corres. $\angle^s$; AB // CD]

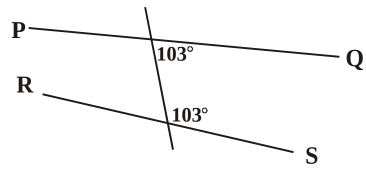
(b) Determine whether the following pairs of lines are parallel or not. Motivate your answer.

(i)



AB // CD, because a pair of corres. $\angle^s$ is equal.

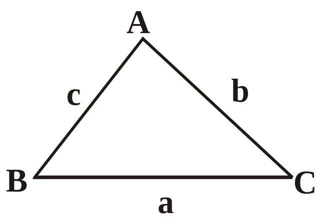
(ii)



PQ is not // to RS, because the co-int $\angle^s$ does not add up to 180°.

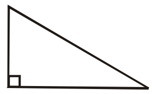
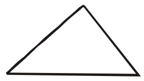
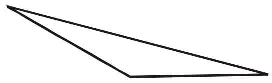
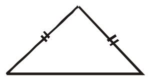
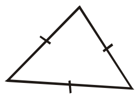
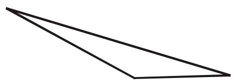
N2 <u>Triangles</u>:

(1) <u>Naming of angles and sides</u>:



A, B and C represent the angles.
a, b and c represent the sides.

(2) Types of triangles:

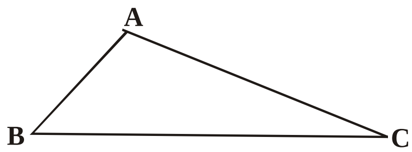
Type of triangle:	Example:	Description:
Right-angled triangle		One angle is equal to 90°.
Acute-angled triangle		All the angles are acute angles.
Obtuse-angled triangle		One of the angles is an obtuse angle.
Isosceles triangle		Two sides are equal in length.
Equilateral triangle		All three sides are equal in length.
Scalene triangle		All three sides have different lengths.

(3) Properties of triangles:

(a) In a triangle, the longest side is always opposite the biggest angle.
(b) In an isosceles triangle, the two angles opposite the equal sides are equal.
(c) In an equilateral triangle, all the angles are equal to 60°.

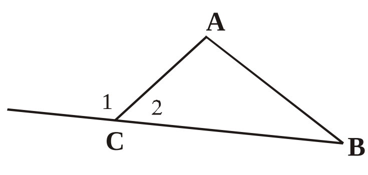
(d) The sum of the interior angles of any triangle is 180°.

$\therefore \hat{A} + \hat{B} + \hat{C} = 180°$



(e) The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

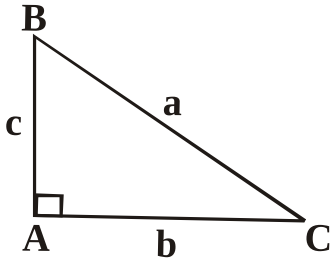
$\therefore \hat{C}_1 = \hat{A} + \hat{B}$



(4) <u>Theorem of Pythagoras</u>:

According to Pythagoras: ‘The square on the hypotenuse is equal to the sum of the squares on the other two sides.’

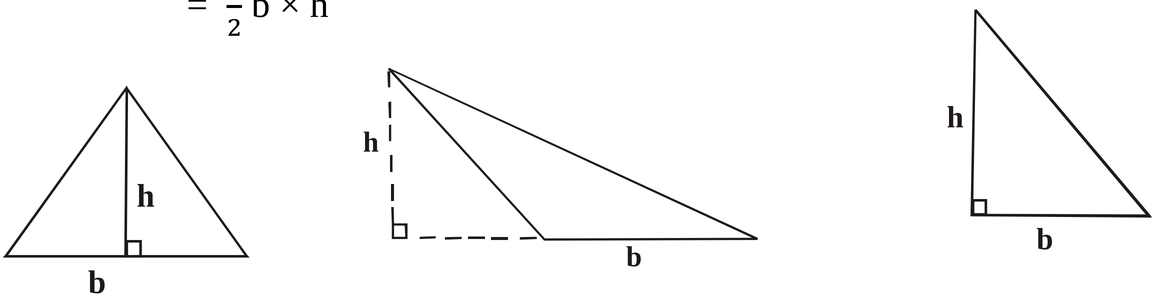
$\therefore \; a^2 = b^2 + c^2$



(5) <u>The area of a triangle</u>:

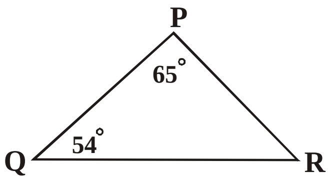
Area Δ $= \frac{1}{2}$ base $\times$ perpendicular height

$= \frac{1}{2}$ b × h



E.g. 2 (a) Calculate the values of x, y and k in each of the following and provide reasons:

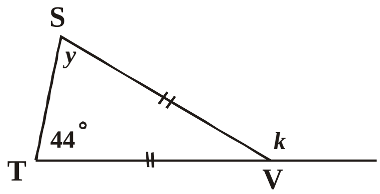
(i)



$65° + 54° + x = 180°$ *[int. $\angle^s$ of Δ]*

$\boldsymbol{x = 61°}$

(ii)



$\boldsymbol{y = 44°}$ *[$\angle^s$ opp equal sides]*

$k = 44° + 44°$ *[ext $\angle$ of Δ]*

$\boldsymbol{k = 88°}$

(b) Calculate the length of the unknown side in the following triangle:

$f^2 = g^2 + e^2$

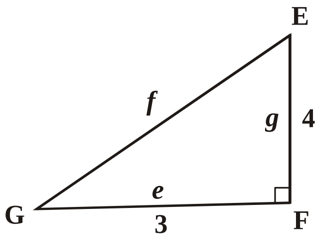
$f^2 = 4^2 + 3^2$

$f^2 = 16 + 9$

$f^2 = 25$

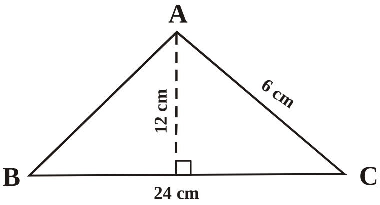
$\sqrt{f^2} = \sqrt{25}$

$\boldsymbol{f = 5}$

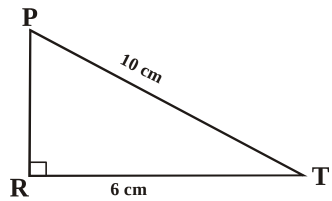


(c) Calculate the area of the following triangles:

(i)



(ii)



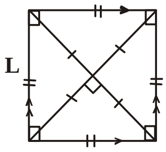
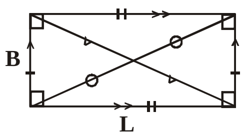
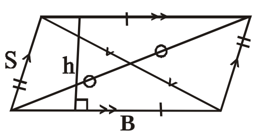
$$Area\ \Delta = \frac{1}{2} b \times h$$
$$= \frac{1}{2}(24)(12)$$
$$= \boldsymbol{144\ cm^2}$$

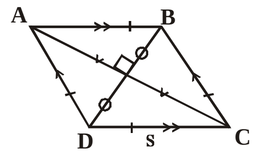
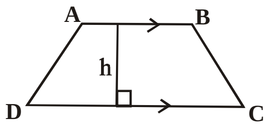
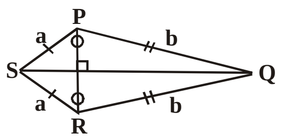
$$r^2 = p^2 + t^2$$
$$100 = 36 + t^2$$
$$100 - 36 = t$$
$$64 = t^2$$
$$\boldsymbol{8 = t}$$

$$Area\ \Delta = \frac{1}{2} b \times h$$
$$= \frac{1}{2}(6)(8)$$
$$= \boldsymbol{24\ cm^2}$$

N3 Quadrilaterals:

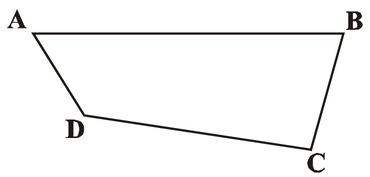
(1) Types of quadrilaterals:

Type of quadrilateral:	**Properties:**	**Perimeter:**	**Area:**
Square 	* All the sides are equal in length. * The opposite sides are parallel. * All the angles are 90°. * The diagonals bisect perpendicularly and bisect the angles. * The diagonals are equal in length.	4L	L^2
Rectangle 	* Opposite sides are parallel and equal in length. * All the angles are 90°. * Diagonals bisect one another. * The diagonals are equal in length.	2L + 2B	L × B
Parallelogram 	* Opposite sides are parallel and equal in length. * Opposite angles are equal. * Diagonals bisect one another.	2B + 2S	B × ⊥h

Rhombus 	* All the sides are equal in length. * The opposite sides are parallel. * Opposite angles are equal. * The diagonals bisect perpendicularly and bisect the angles.	4s	$\frac{1}{2}$ AC $\times$ BD or B $\times$ $\perp$h
Trapezium 	* Only one pair of opposite sides is parallel.	AB+BC+CD+DA	$\frac{1}{2}$ h $\times$ (AB + CD)
Kite 	* Pairs of adjacent sides are equal. * One pair of opposite angles is equal. * The diagonals are perpendicular to one another and the longest diagonal bisects the shorter diagonal.	2a + 2b	$\frac{1}{2} \times$ SQ $\times$ PR

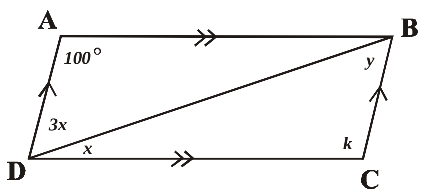
(2) The sum of the interior angles of a quadrilateral is 360°.

$\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$

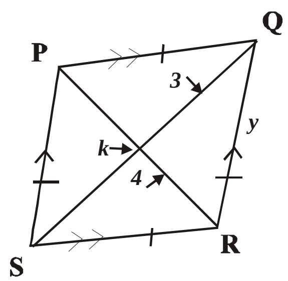


E.g. 3 (a) Calculate, with reasons, the values of x, y and k:

(i)



(ii)



(i)

$100° + 3x + x = 180°$ *[co-int ∠^{s}; AB // CD]*

$4x = 80°$

$\boldsymbol{x = 20°}$

$3x = \boldsymbol{y = 60°}$ *[alt ∠^{s}; AD // BC]*

$\boldsymbol{k = 100°}$ *[area ∠^{s} of parm]*

(ii)

$\boldsymbol{k = 90°}$

[Diagonals bisect ⊥]

$y^2 = 3^2 + 4^2$ *[Pythagoras]*

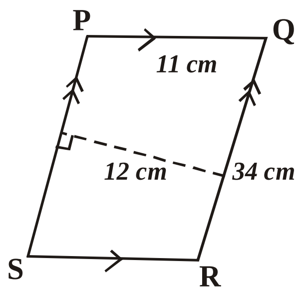
$y^2 = 9 + 16$

$y^2 = 25$

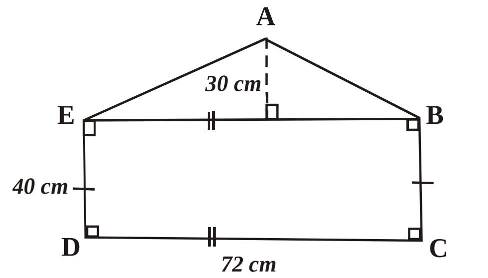
$\boldsymbol{y = 5}$

(b) Determine the areas of the following triangles:

(i)



(ii)



Area PQRS $= b \times \perp h$

$= 34 \times 12$

$= \mathbf{408\ cm^2}$

Area ABCDE $= [L \times B] + [\frac{1}{2} \times B \times \perp h]$

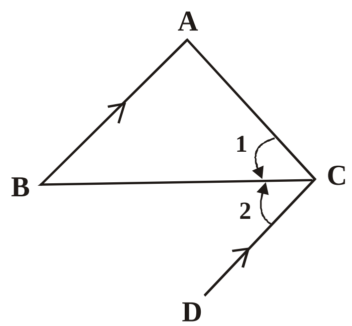
$= [72 \times 40] + [\frac{1}{2} \times 72 \times 30]$

$= [2\ 880] + [1\ 080]$

$= \mathbf{3\ 960\ cm^2}$

N4 <u>More applications</u>:

E.g. 4 (1) Prove that $\triangle ABC$ *is an isosceles* Δ *if AB // CD and* $\hat{C}_1 = \hat{C}_2$:



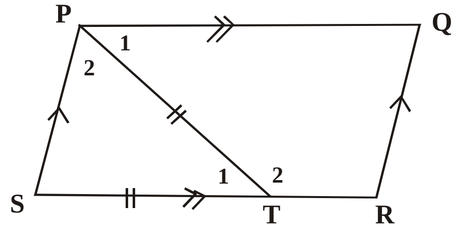
$\hat{C}_2 = \hat{B}$ *[alt* $\angle^s$*; PT // ST]*

but $\hat{C}_1 = \hat{C}_2$ *[given]*

$\therefore\ \hat{C}_1 = \hat{B}$

$\therefore\ AB = AC$ *[sides opp equal angles]*

(2) PQRS is a parallelogram, with PT = ST.
If $\hat{S} = x$*, prove that* $\hat{P}_1 = 180° - 2x$.



$\hat{P}_2 = x$ *[angles opp equal sides]*

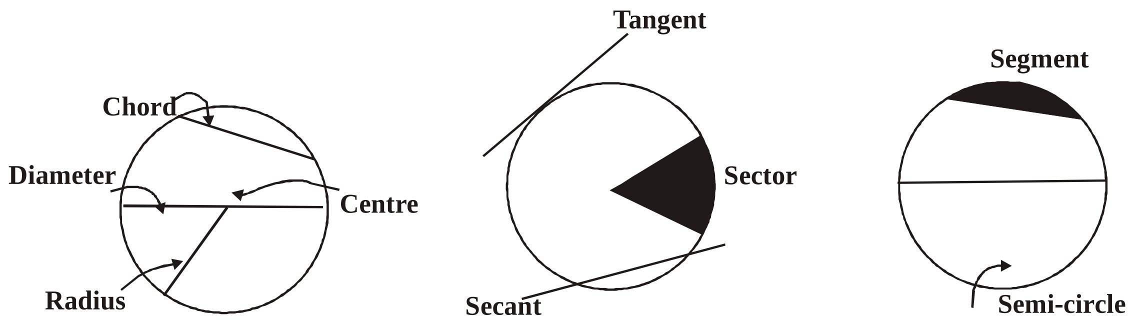
$\hat{T}_1 = 180° - 2x$ *[int* $\angle^s$ *of* Δ*]*

but $\hat{T}_1 = \hat{P}_1$ *[alt* $\angle^s$*; PQ // SR]*

$\therefore\ \hat{T}_1 = 180° - 2x$

N5 **Circles:**

(1) Terminology:



(2) Area and circumference:

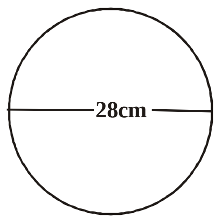
Circumference $= 2 \times \pi \times r$ and Area $= \pi \times r^2$

Remember: $\pi = \frac{22}{7}$ and Diameter (d) $= 2 \times$ radius (r)

E.g. 5 (a) Determine the area and circumference of the following circle:

$$\begin{aligned} \textit{Circumference} &= 2\pi r \\ &= 2 \times \frac{22}{7} \times 14 \\ &= \mathbf{88\ cm} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \frac{22}{7} \times (14)^2 \\ &= \frac{22}{7} \times 196 \\ &= \mathbf{616\ cm^2} \end{aligned}$$



(b) The area of the circle is 201 mm². Determine the length of the radius to the nearest whole number.

$$\begin{aligned} \text{Area} &= \pi r^2 \\ 201 &= \frac{22}{7} \times r^2 \\ 201 \times 7 &= 22 \times r^2 \\ 1\,407 &= 22r^2 \\ \frac{1\,407}{22} &= r^2 \\ 63{,}95\ldots &= r^2 \\ \mathbf{8} &= \mathbf{r} \end{aligned}$$