Wes-Kaapse Regering
Onderwys

OVERBERG EDUCATION DISTRICT

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS
PAPER 2

SEPTEMBER 2016

MARKS: 150

TIME: 3 hours

This paper consist of 12 pages and a formula sheet

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5. Answers only will not necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

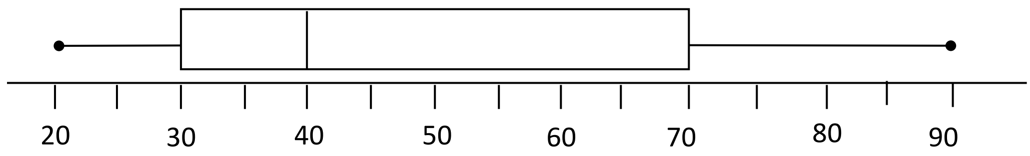
8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

The box and whisker diagram below shows the marks (out of 90) obtained in a Mathematics test by a class of nine learners.



1.1 Comment on the skewness of the data. (1)

1.2 Write down the interquartile range of the marks obtained. (2)

1.3 If the learners had to obtain 30 marks to pass the test, estimate the percentage of the class that passed the test. (2)

1.4 In ascending order, the second and third marks are the same, the sixth mark is 58, the seventh mark is 66 and the eighth mark is 74. The average mark for this test is 49.

					58	66	74	

Fill in the marks of the remaining learners in ascending order.(on answersheet) (7)

[12]

QUESTION 2

The table below shows the amount of time (in hours) that learners aged between 12 and 16 spent playing sport during school holidays

Time (hours)	**Cumulative Frequency**
$0 \leq t < 20$	30
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	167
$100 \leq t < 120$	172

2.1 Draw an ogive (cumulative frequency curve) on the answersheet provided to represent the data above. (4)

2.2 Write down the modal class of the data. (1)

2.3 How many learners played sport during the school holidays, according to the data above?(1)

2.4 Use the ogive (cumulative frequency curve) to estimate the number of learners who played sport more than 60% of the time. (2)

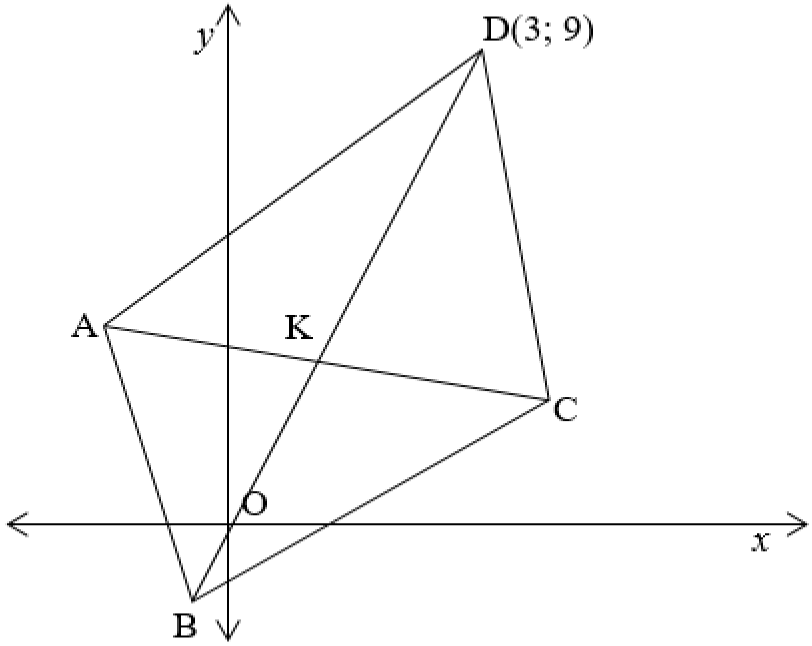
2.5 Estimate the mean time (in hours) that learners spent playing sport during the school holidays. (4)

[12]

QUESTION 3

3.1 In the diagram, A, B, C and D are the vertices of a rhombus. $x+3y=10$

The equation of AC is $x+3y=10$.



3.1.1 Show that the equation of BD is $3x-y=0$. (3)

3.1.2 Calculate the coordinates of K, the point of intersection of AC and BD. (4)

3.1.3 Determine the coordinates of B. (3)

3.1.4 If C is the point $(5;y)$, determine the value of y. (4)

3.2 P(-3;2) and Q(5;8) are two points in a Cartesian plane.

3.2.1 Calculate the gradient of PQ. (2)

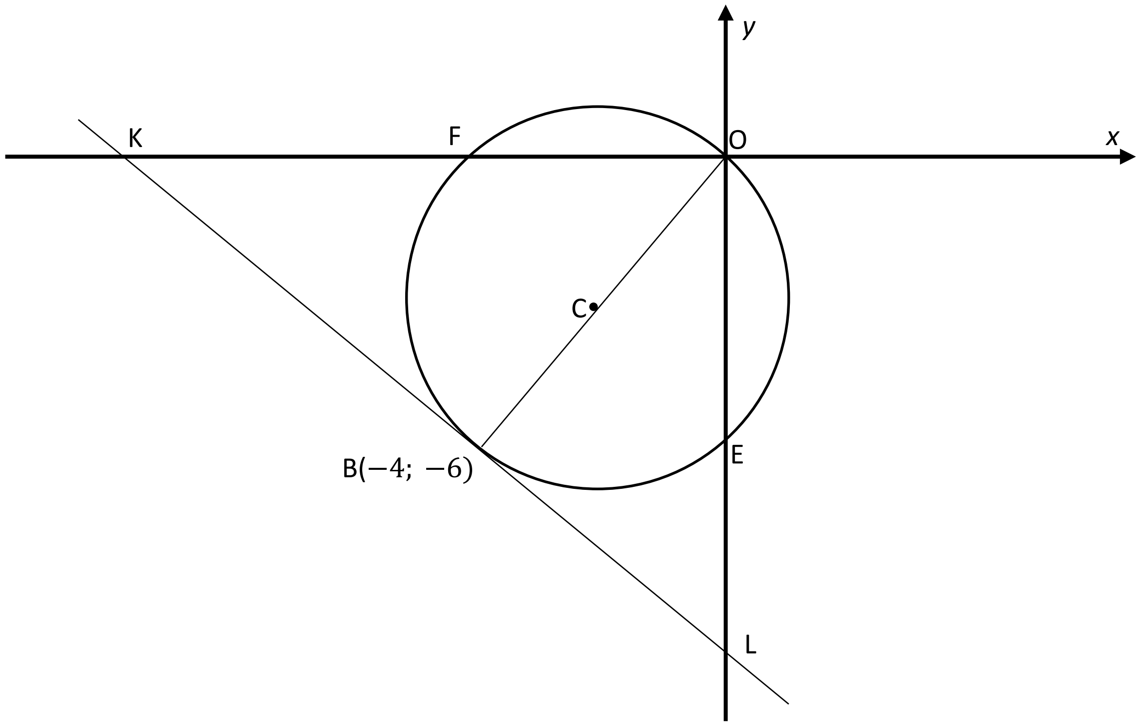
3.2.2 Calculate the angle that PQ forms with the positive x-axis, correct to one decimal place. (2)

3.2.3 Determine the equation of the straight line parallel to PQ that intercepts the *x*-axis at (8; 0). (3)

[21]

QUESTION 4

In the diagram, the circle, having centre at C, passes through the origin, O, and intersects the *x*-axis at F and the *y*-axis at E. The tangent to the circle at B(−4 ; −6) intersects the *x*-axis at K and the *y*-axis at L.



4.1 Calculate the coordinates of C.. (2)

4.2 Calculate the length of the radius of the circle. (2)

4.3 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^r$. (2)

4.4 Why is ΔOBL a right angled triangle? (1)

4.5 Determine the equation of the tangent KL in the form $y=mx+c$. (4)

4.6 Calculate the coordinates of E. (4)

4.7 Determine the ratio of the following in its simplest form:

$$\frac{\text{area } \Delta\text{OBE}}{\text{area } \Delta\text{OBL}}$$ (4)

[19]

QUESTION 5

5.1 Simplify as far as possible $1-\sin^2\theta+3-\cos^2\theta$. (3)

5.2 Simplify WITHOUT the use of a calculator:

$$\frac{\cos 160^\circ \tan 200^\circ}{2\sin(-10^\circ)}$$ (6)

5.3 Prove that $\dfrac{\cos^2 x\sin^2 x+\cos^4 x}{1-\sin x}=1+\sin x$ (4)

5.4 Consider $\cos(x+45^\circ)\cos(x-45^\circ)$.

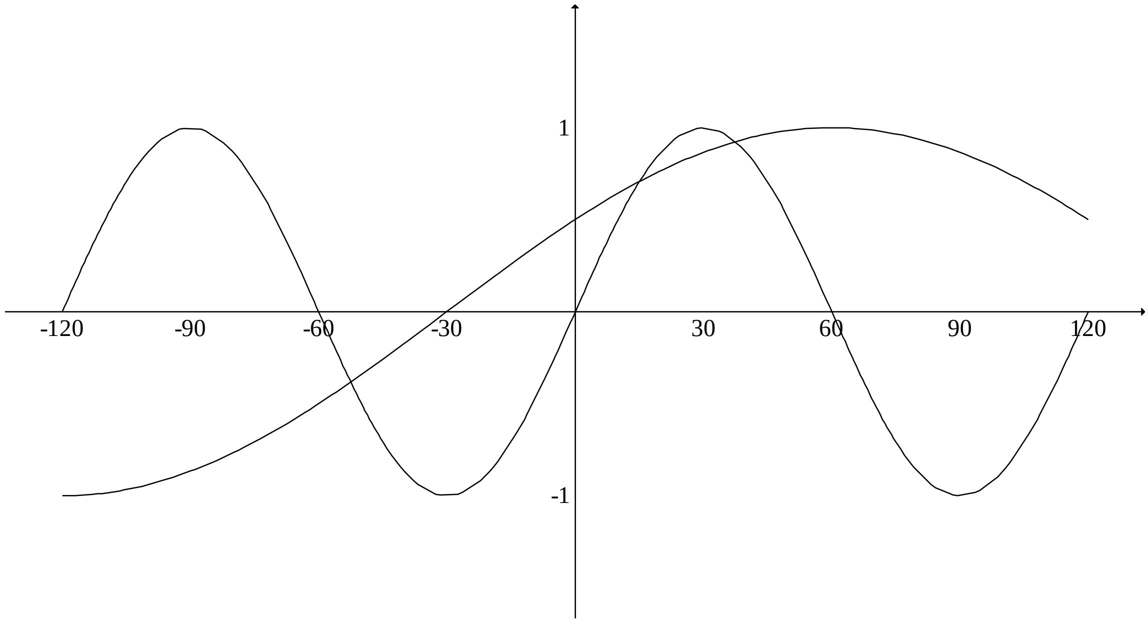
5.4.1 Show that $\cos(x+45^\circ)\cos(x-45^\circ)=\dfrac{1}{2}\cos 2x$ (4)

5.4.2 Hence, determine a value of x in the interval $0^\circ \le x \le 180^\circ$ for which $\cos(x+45^\circ)\cos(x-45^\circ)$ is a minimum. (3)

[20]

QUESTION 6

In the diagram, the functions $f(x)=\cos(x-60°)$ and $g(x)=\sin 3x$ are drawn for $x \in [-120°\,;120°]$.



6.1 Write down the period of $g(x)$. (1)

6.2 Write down the amplitude of $f(x)$. (1)

6.3 Give the range of $f(x)-1$. (1)

6.4 Determine the *x* value(s), $x \in [-120°\,;120°]$ where the two graphs intersect. (6)

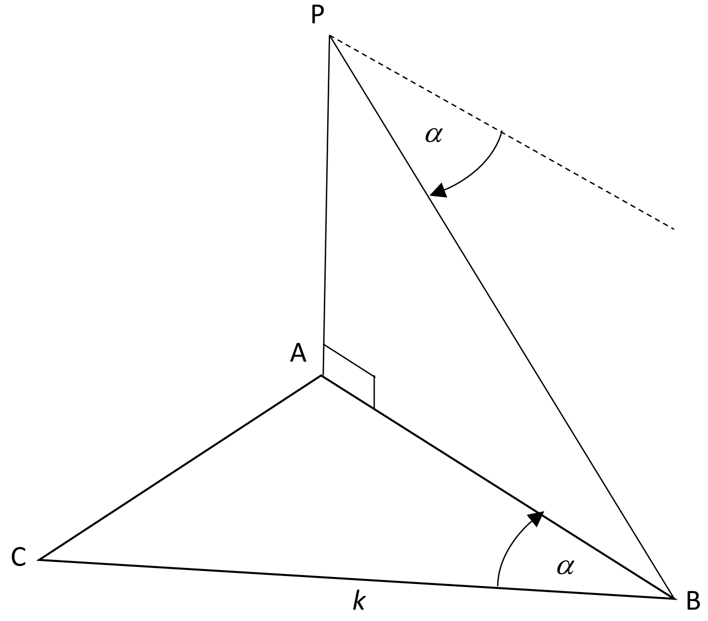
6.5 For which values of x, $x \epsilon [0°; 120°]$, will $g(x-5°) > f(x-5°)$? (3)

6.6 The graph h is obtained when the graph of f is shifted 45° to the left. Write down the equation of h in its simplest form. (2)

[14]

QUESTION 7

In the diagram, P is a point directly above A of ΔABC. A, B and C are points in the same plane. BC = k metres and $A\hat{B}C = \alpha$. The angle of depression of B, as seen from P, is α and the area of ΔABC = t metres^2.



7.1 Write $A\hat{B}P$ in terms of α. (1)

7.2 Show that $AB = \dfrac{2t}{k\sin\alpha}$. (2)

7.3 Prove that $PB = \dfrac{4t}{k\sin 2\alpha}$ (4)

[7]

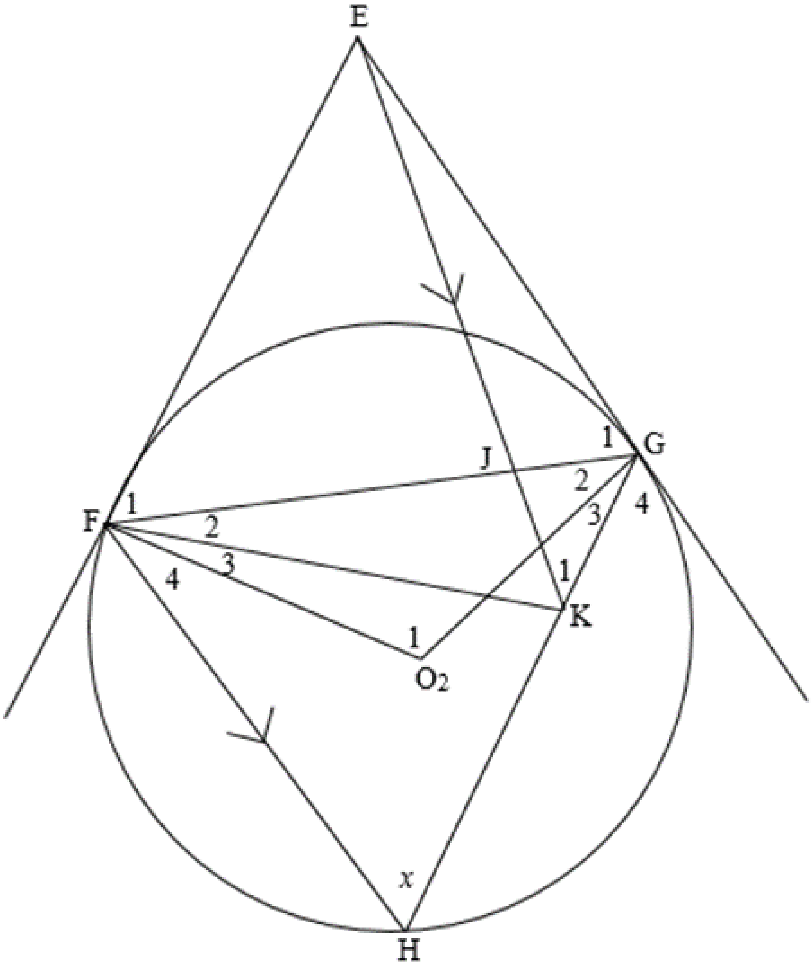
Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 Complete the following:

The opposite angles of a cyclic quadrilateral are … (1)

8.2 In the diagram, EF and EG are tangents to circle with centre O. FH || EK, EK intersects FG at J and meets GH at K. $\hat{H} = x$



Prove that …

8.2.1 FOGE is a cyclic quadrilateral. (5)

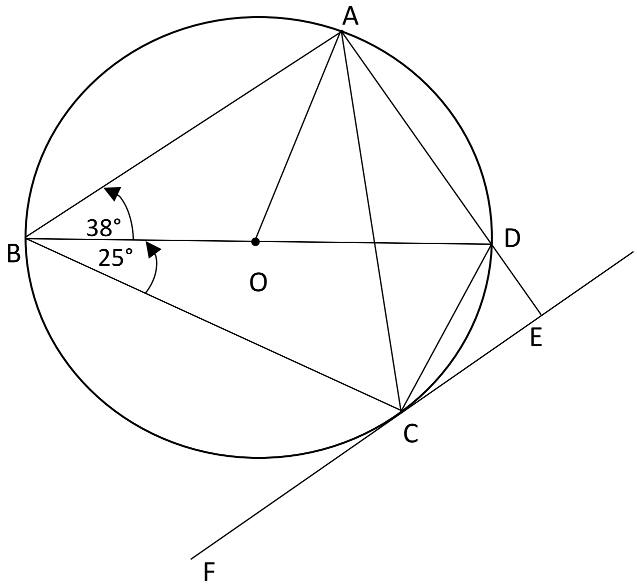
8.2.2 EG is a tangent to the circle GJK. (5)

8.2.3 $\angle FEG = 180° - 2x$ (3)

[14]

QUESTION 9

9.1 In the diagram, ABCD is a cyclic quadrilateral in the circle, having centre O. Tangent FC meets AD produced in E. Radius AO and chord AC are drawn. $A\hat{B}O = 38°$ and $C\hat{B}D = 25°$.



Calculate, with reasons, the size of each of the following angles:

9.1.1 $B\hat{A}D$ (2)

9.1.2 $A\hat{O}D$ (2)

9.1.3 $C\hat{D}E$ (2)

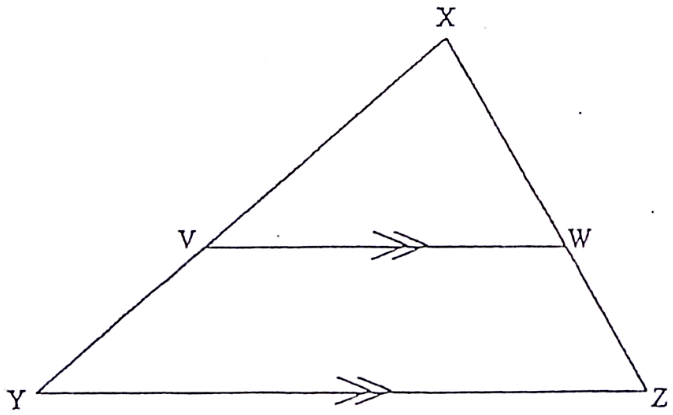
9.1.4 $D\hat{C}E$ (2)

9.1.5 $A\hat{C}D$ (2)

[10]

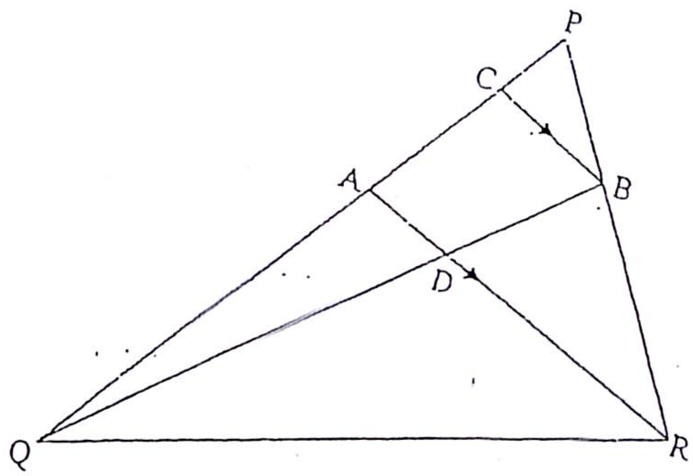
QUESTION 10

10.1 Use that diagram on your diagram sheet to prove that theorem which states that if $VW \parallel YZ$, then $\frac{XV}{VY} = \frac{XW}{WZ}$.



(6)

10.2 In ΔPQR below, B lies on PR such that 2PB=BR. A lies on PQ such that PA:PQ=3:8. CB is drawn parallel to AR.



10.2.1 Write down the value of $\frac{area \Delta PRA}{area \Delta QRA}$. (2)

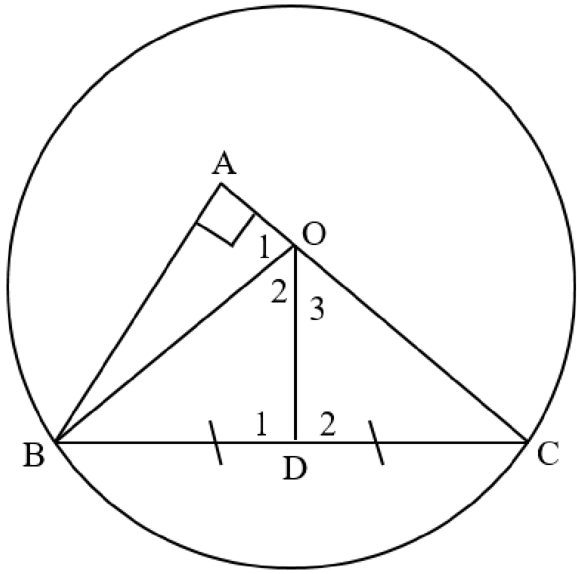
10.2.2 Calculate the value of the ratio $\frac{BD}{BQ}$.

Show all workings to support your answer. (4)

[12]

QUESTION 11

Given O is the centre of the circle. BA $\perp$ AC. D is the midpoint of BC.



11.1 Prove that: ΔABC ||| ΔDOC (4)

11.2 Show that: $OC = \frac{DC.BC}{AC}$ (1)

11.3 Given: DC = 15 cm and AB = 18 cm.

Calculate the length of OC, rounded off to one decimal unit. (4)

[9]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ M$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$oppervlakte\ \Delta \mathrm{ABC} = \frac{1}{2} ab.\sin \mathrm{C}$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$