

**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2018**

10612

**MATHEMATICS
PAPER 2**

TIME: 3 hours

MARKS: 150

15 pages, 1 information sheet and a 21 page answer book

MATHEMATICS: Paper 2

1061E



10612E

X05



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(Paper 2)

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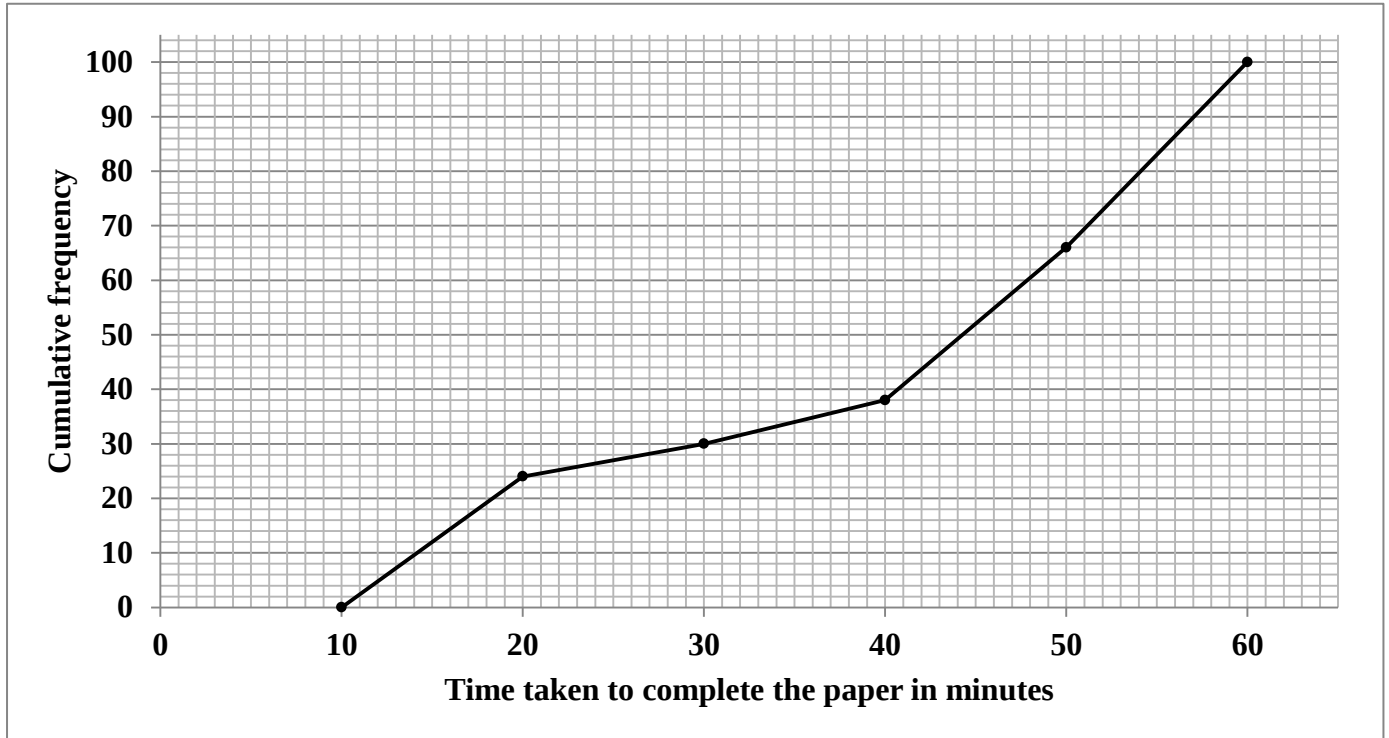
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

In a Mathematics competition learners were expected to answer a multiple choice question paper. The time taken by the learners to the nearest minute to complete the paper, was recorded and data was obtained. The cumulative frequency graph representing the time taken to complete the paper is given below.



An incomplete frequency table for the data is given below.

Time taken to complete the paper in minutes	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency	a	6	8	28	34

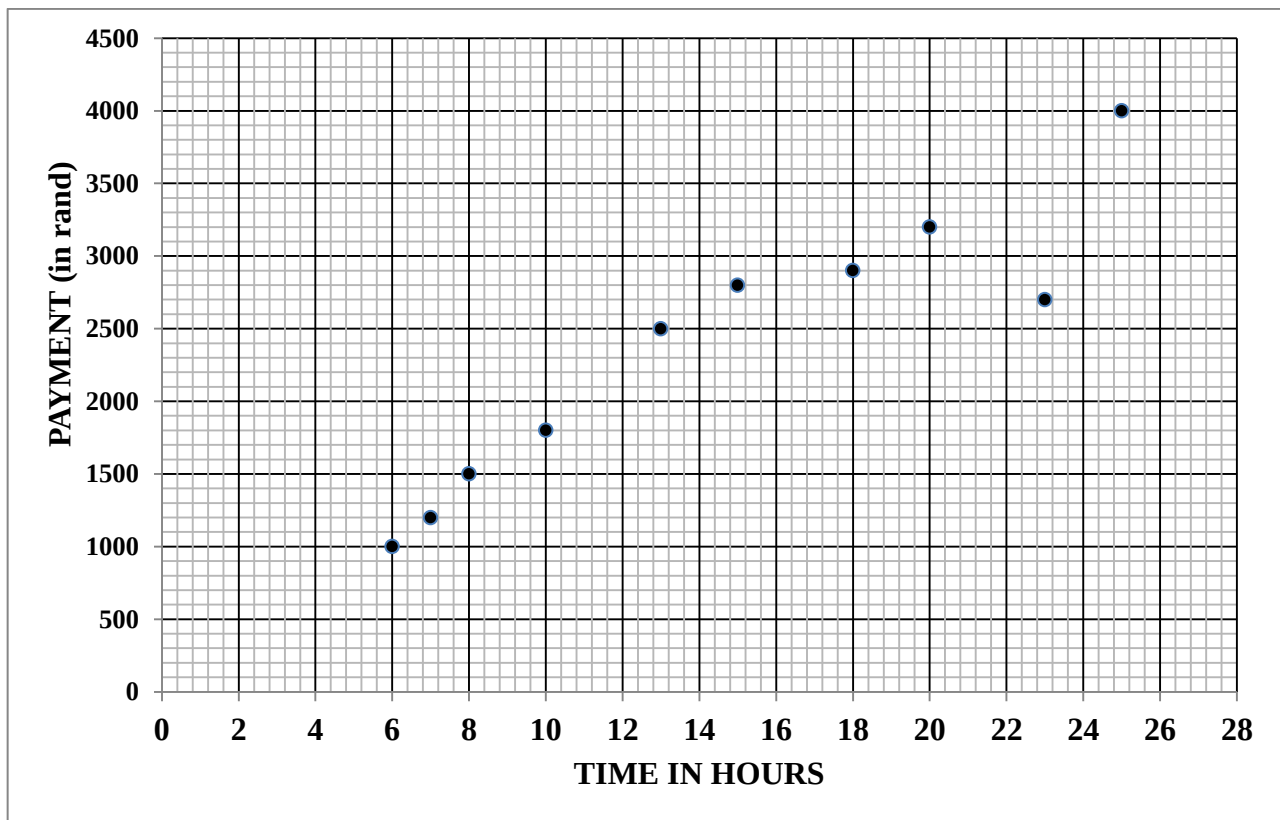
- 1.1 Determine the value of a in the frequency table. (2)
- 1.2 How many learners wrote the paper? (1)
- 1.3 Identify the modal class of the data. (1)
- 1.4 Calculate:
 - 1.4.1 The estimated mean time, in minutes, taken to complete the paper (3)
 - 1.4.2 The number of learners that took longer than 35 minutes to complete the paper (2)

[9]

QUESTION 2

A group of students did some part-time work for a company. The number of hours that the students worked and the payment (in rand) received for the work done is shown in the table below. The scatter plot is drawn for the data.

Number of hours worked	6	7	8	10	13	15	18	20	23	25
Payment (in rand)	1000	1200	1500	1800	2500	2800	2900	3200	2700	4000

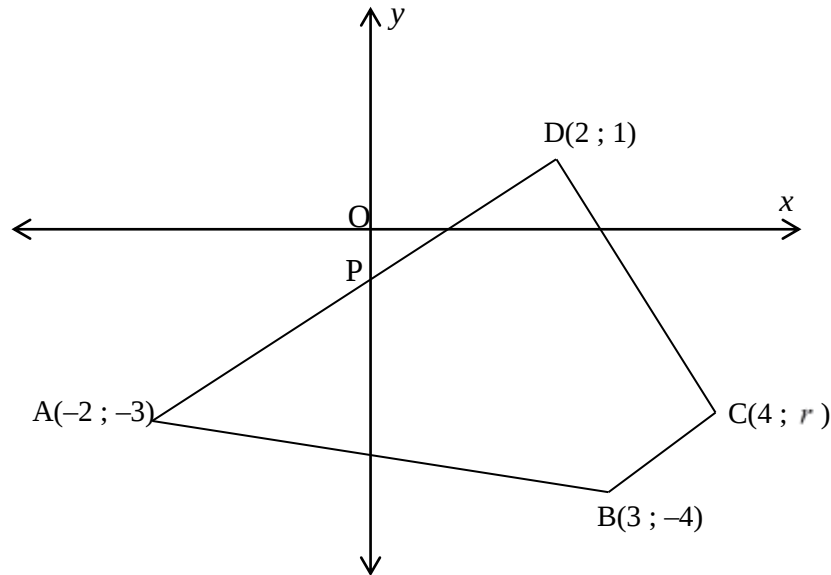


- 2.1 Calculate the standard deviation of the number of hours worked. (1)
- 2.2 Determine the number of hours that a student needed to work in order to receive a payment that was more than one standard deviation above the mean. (3)
- 2.3 Determine the equation of the least squares regression line of the data. (3)
- 2.4 Mapula who worked for 11,5 hours was omitted from the original data. Calculate the possible amount that the company has to pay Mapula. (2)
- 2.5 Use the scatter plot to identify an outlier and give a possible reason for this point to be an outlier. (2)

[11]

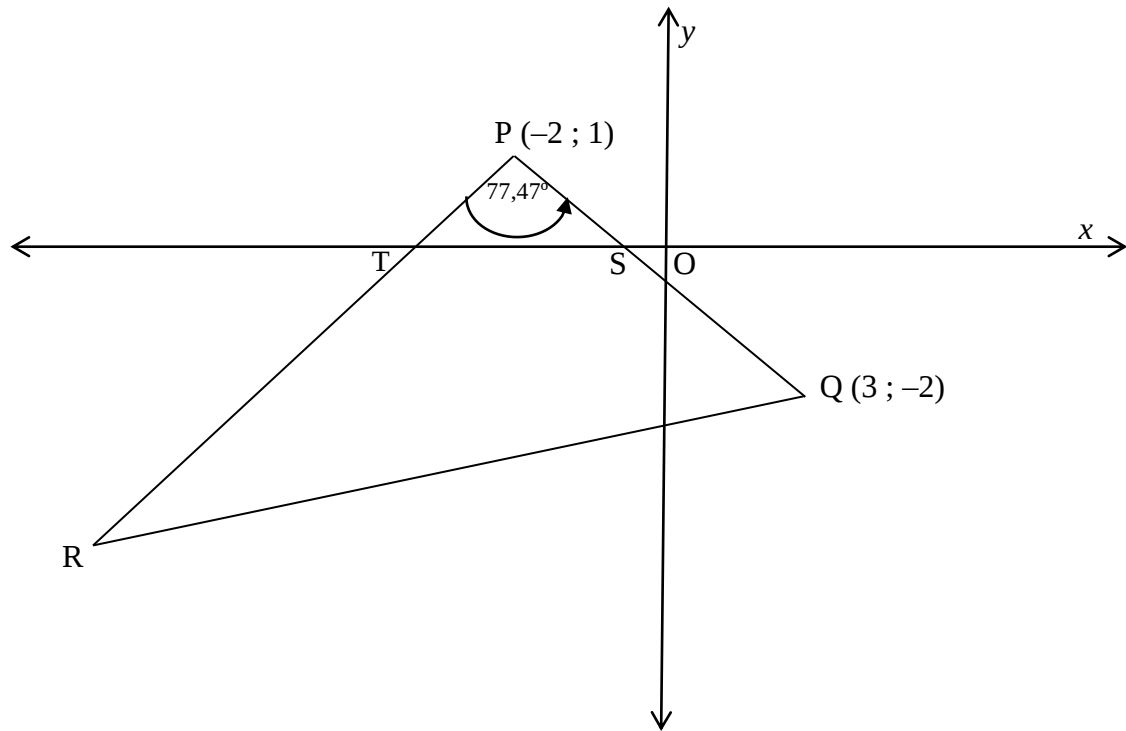
QUESTION 3

- 3.1 In the diagram below, points $A(-2 ; -3)$, $B(3 ; -4)$, $C(4 ; r)$ and $D(2 ; 1)$ are the vertices of quadrilateral ABCD. P is the midpoint of line AD.



- 3.1.1 Calculate the value of r if $AD \parallel BC$. (4)
- 3.1.2 What type of quadrilateral is ABCD? (1)
- 3.1.3 Determine the coordinates of P. (2)
- 3.1.4 Prove that $BP \perp AD$. (2)
- 3.1.5 Determine the equation of the circle passing through PBA in the form $(x - a)^2 + (y - b)^2 = r^2$. (5)
- 3.1.6 Calculate the maximum radius of the circle having equation $x^2 + y^2 - 2x \cos \theta - 4y \cos \theta = -2$ for any value of θ . (5)

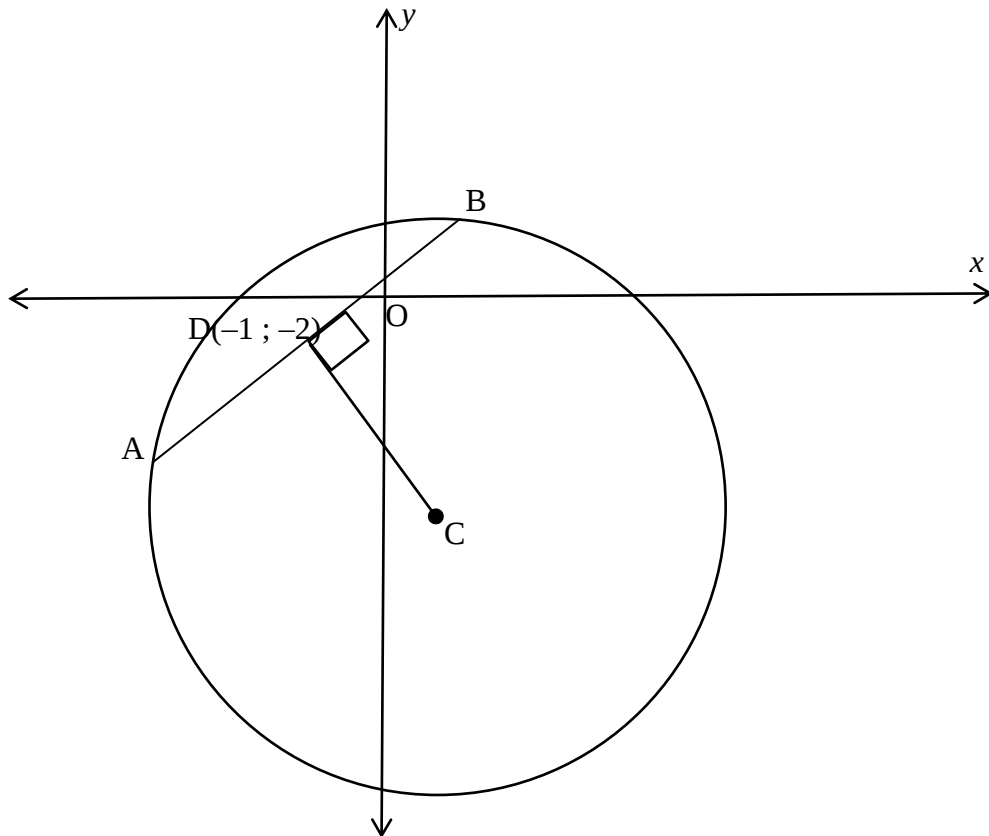
- 3.2 In the diagram below, points $P(-2 ; 1)$ and $Q(3 ; -2)$ are given and R is a point in the third quadrant. PQ and PR cut the x -axis at S and T respectively.
 $\hat{QPR} = 77,47^\circ$.



- 3.2.1 Determine the equation of line PQ in the form $ax + by + c = 0$ (3)
- 3.2.2 Determine the equation of PR in the form $y = mx + c$. (6)
- [28]**

QUESTION 4

In the diagram below, AB is a chord of the circle with centre C. D(-1 ; -2) is the midpoint of AB. $DC \perp AB$. The equation of the circle is $x^2 + y^2 + 6y = 4x + 12$.



- 4.1 Determine the coordinates of C. (3)
 - 4.2 Determine the radius of the circle. (1)
 - 4.3 Calculate the length of AB. (5)
 - 4.4 Calculate the area of $\triangle ABC$. (3)
- [12]**

QUESTION 5

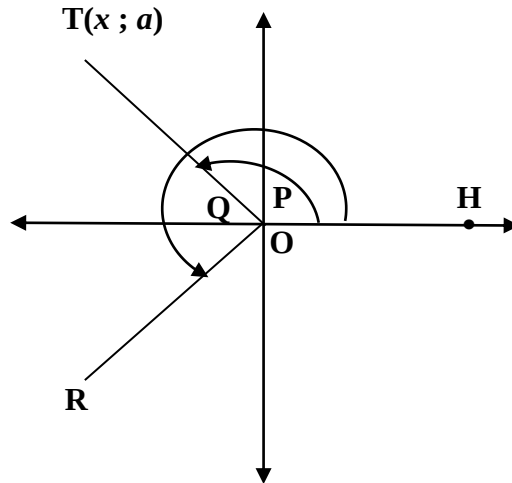
5.1 Simplify the following expression to a single trigonometric function.

$$\frac{\sin x \cdot \sin(90^\circ + y) - \cos x \cdot \sin(180^\circ + y)}{\cos x \cdot \cos(y - 360^\circ) + \sin(-x) \sin y} \quad (6)$$

5.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

5.2.1 Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ (2)

5.2.2 In the diagram, T is a point such that $\hat{HOT} = P$ and $\sin P = a$. T is reflected about the x-axis to R such that $\hat{HOR} = Q$



(a) Determine the coordinates of T in terms of a . (2)

(b) Write down the coordinates of R in terms of a . (2)

(c) Calculate $\cos(P + Q)$. (2)

(d) Hence, show that $P + Q = 360^\circ$. (1)

5.3 Given: $\cos \theta = d$

5.3.1 Write down the values of d such that $\cos \theta$ is defined. (2)

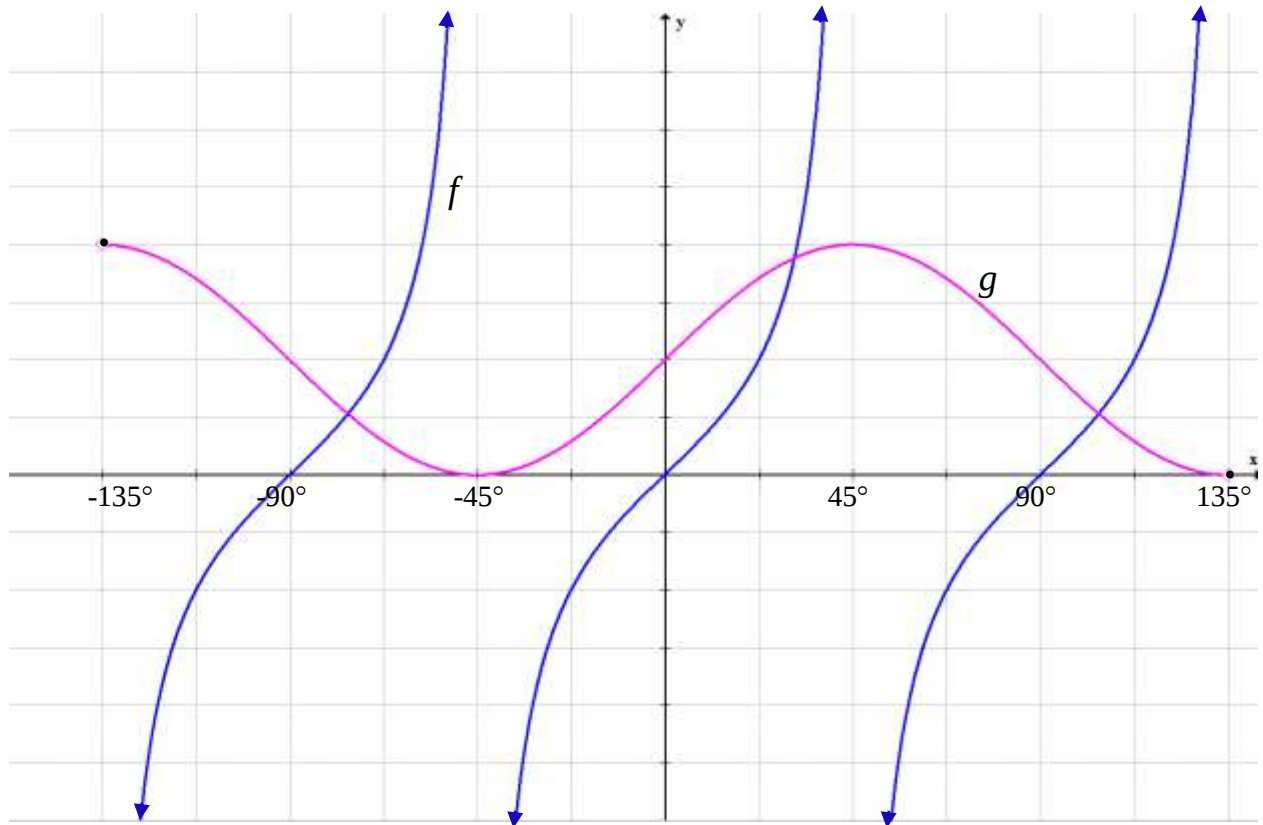
5.3.2 Determine the general solution for θ if :

$$\cos \theta = \frac{1}{\cos \theta} + \frac{5}{6} \quad (6)$$

[23]

QUESTION 6

The functions $f(x) = \tan 2x$ and $g(x) = 1 + \sin 2x$ are sketched for $x \in [-135^\circ; 135^\circ]$.

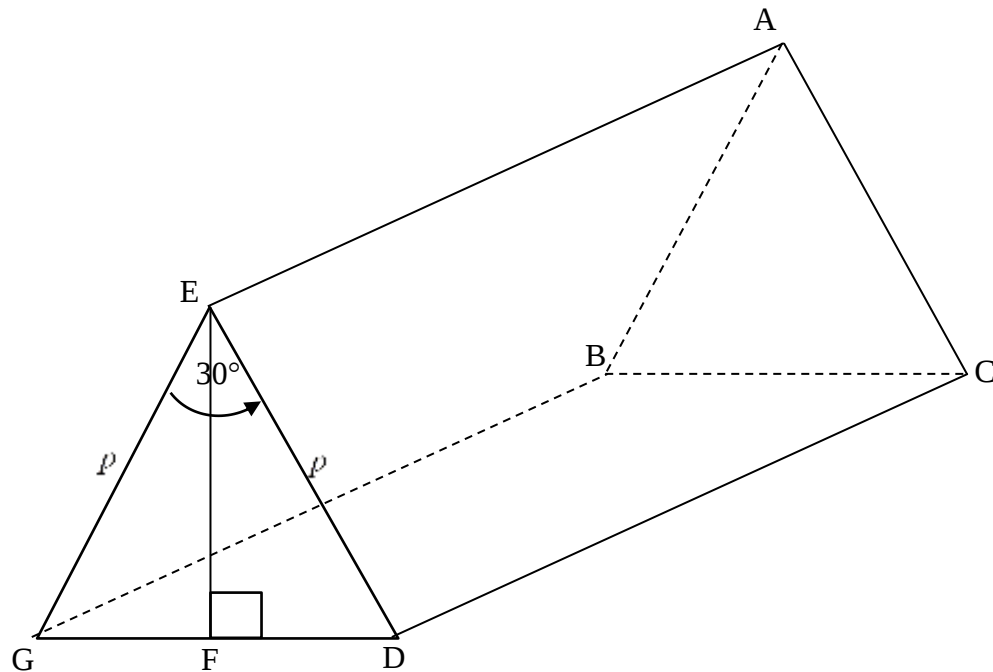


- 6.1 Write down the equation of the asymptote in the interval $x \in [-135^\circ; 0^\circ]$. (1)
- 6.2 If $h(x) = \frac{\sin x - 2\sin^3 x}{2\sin^2 x \cdot \cos x}$, determine h in terms of f . (4)
- 6.3 Determine the equation of p in its simplest form, if graph g is translated by moving the y -axis 45° to the right. (3)
- 6.4 Determine the values of x for which $(\tan 2x)(-1 - \sin 2x) \leq 0$ for $x \in [-135^\circ; 0^\circ]$. (3)

[11]

QUESTION 7

The given figure represents a roof in the form of a triangular prism. The beams EG and ED have length p metres. $EF \perp GD$ and $\hat{GED} = 30^\circ$.



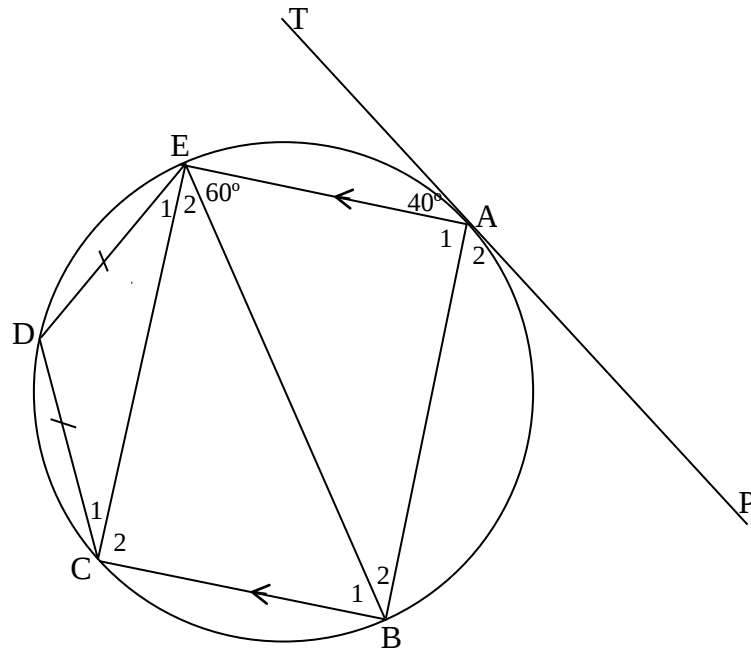
Without using a calculator:

- 7.1 Prove that $GD^2 = p^2(2 - \sqrt{3})$. (3)
- 7.2 Hence, determine the value of CD in terms of p , if $\hat{CGD} = 60^\circ$. (3)
- [6]**

GIVE REASONS FOR ALL STATEMENTS AND CALCULATIONS IN QUESTIONS 8, 9, 10 AND 11.

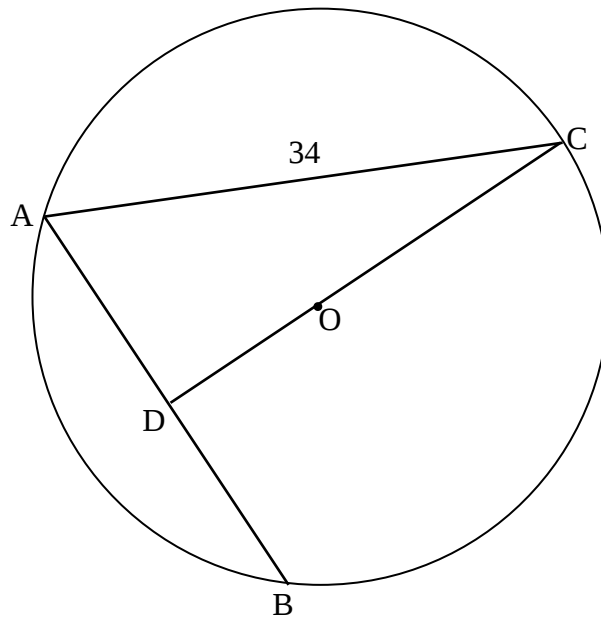
QUESTION 8

In the diagram below, TAP is a tangent to circle ABCDE at A. $AE \parallel BC$ and $DC = DE$.
 $\angle TAE = 40^\circ$ and $\angle AEB = 60^\circ$.



- 8.1 Identify TWO cyclic quadrilaterals. (2)
- 8.2 Determine, with reasons, the size of the following angles:
- 8.2.1 $\hat{B}_2$ (2)
- 8.2.2 $\hat{B}_1$ (2)
- 8.2.3 $\hat{D}$ (2)
- 8.2.4 $\hat{E}_1$ (3)

- 8.3 In the diagram below, radius CO is produced to bisect chord AB at D.
CA = 34 mm and AB = 40 mm

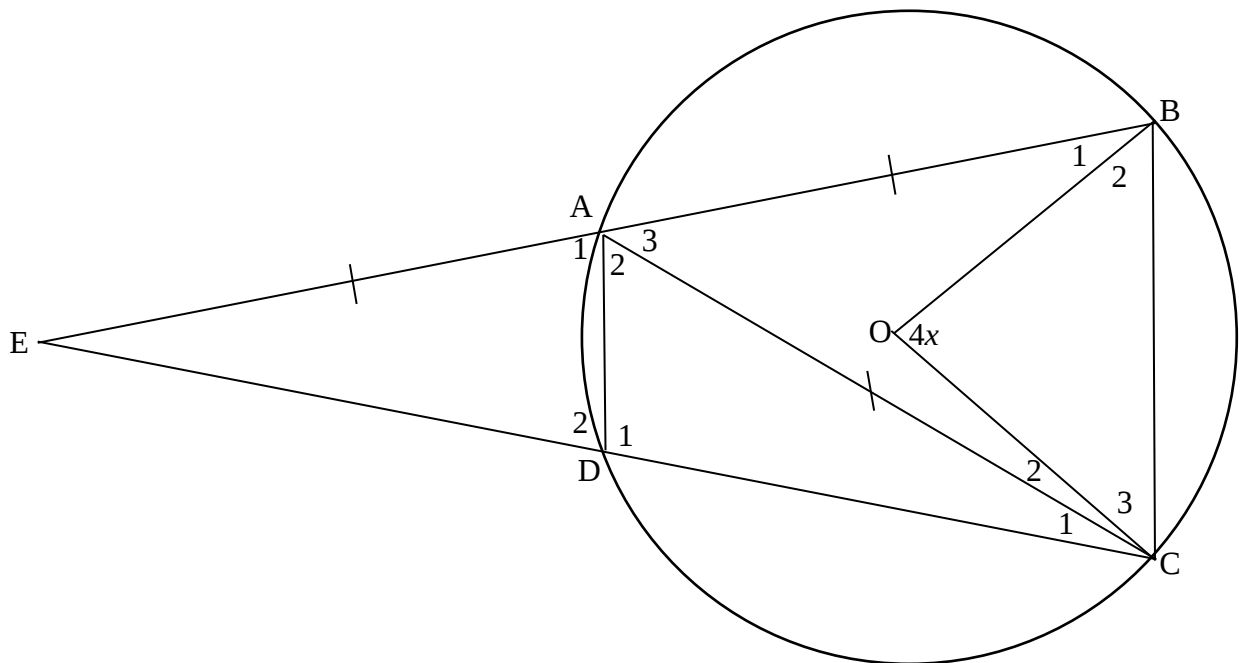


Calculate the size of $\hat{C}$.

(4)
[15]

QUESTION 9

In the diagram below, O is the centre of the circle. ABCD is a cyclic quadrilateral. BA and CD are produced to intersect at E such that $AB = AE = AC$.



Determine in terms of x :

9.1 $\hat{B}_2$ (2)

9.2 $\hat{E}$ (5)

9.3 $\hat{C}_2$ (3)

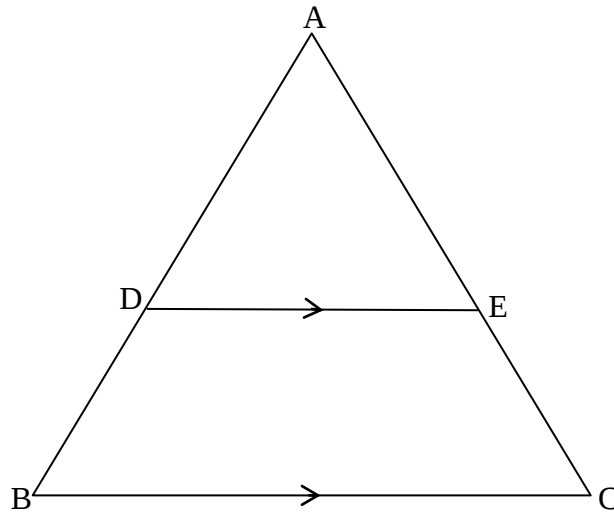
9.4 If $\hat{E} = \hat{C}_2 = x$, prove that ED is a diameter of circle AED. (4)
[14]

QUESTION 10

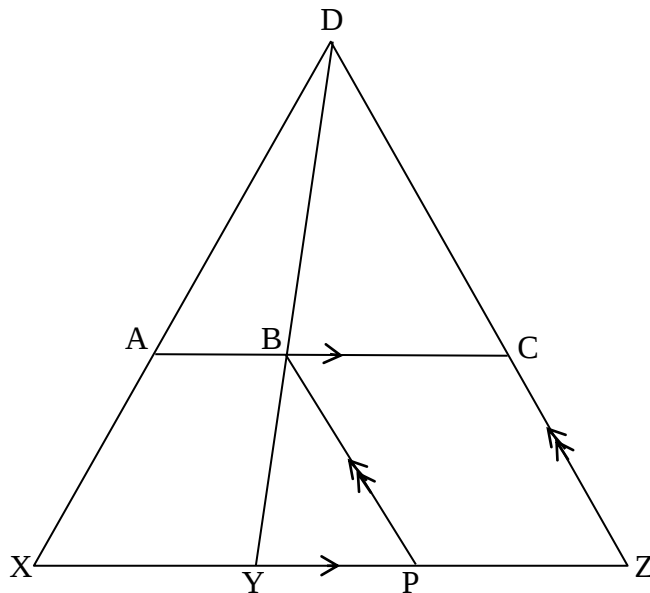
- 10.1 In $\triangle ABC$ below, D and E are points on AB and AC respectively such that $DE \parallel BC$.

Prove the theorem that states that $\frac{AD}{DB} = \frac{AE}{EC}$.

(6)



- 10.2 In $\triangle DXZ$ below, $AC \parallel XZ$ and $BP \parallel DZ$. DY is drawn to intersect AC at B.

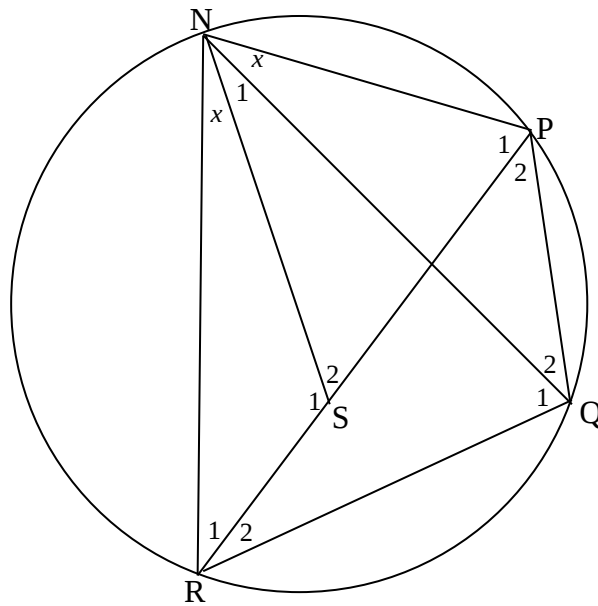


Prove that: $\frac{BC}{YZ} = \frac{DA}{DX}$

(5)
[11]

QUESTION 11

In the diagram below, NPQR is a cyclic quadrilateral with S a point on chord PR. N and S are joined and $\hat{RNS} = \hat{PNQ} = x$.



Prove that:

11.1 $\triangle NSR \parallel \triangle NPQ$ (3)

11.2 $\triangle NQR \parallel \triangle NPS$ (3)

11.3 $NR.PQ + NP.QR = NQ.PR$ (4)
[10]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$