OVERBERG
EDUCATION DISTRICT

COMMON PAPER

GRADE 12

MATHEMATICS P2
SEPTEMBER 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages,1 information sheet and an ANSWER BOOK of 20 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided. Write your name and class in the space provided and hand it in.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

10. An information sheet with formulae is included at the end of the question paper.

11. Write neatly and legibly.

QUESTION 1

90 learners wrote a test out of 60, and their results are shown in the table below.

Mark	Frequency
$0 < x \leq 10$	2
$10 < x \leq 20$	9
$20 < x \leq 30$	21
$30 < x \leq 40$	32
$40 < x \leq 50$	19
$50 < x \leq 60$	7

1.1 Complete the cumulative frequency table in the **answer book**. (2)

1.2 Draw a cumulative frequency curve (ogive) for the data above. (3)

1.3 An award was given to all learners who obtained above 45 out of 60. How many learners received an award? (2)

[7]

QUESTION 2

2.1 Two learners, Dalvon and Chumani, exercised hard one afternoon. They both recorded their pulse rates, P, and time, t minutes, after they were done exercising. Dalvon's results are shown in the table below.

Time in minutes (*t*)	0,5	1,0	1,5	2,0	3,0	4,0	5,0
Dalvon's pulse rate (*P*)	125	113	104	95	82	85	72

2.1.1 Calculate the equation of the least squares regression line for Dalvon's results in the form $y = a + bx$. (3)

2.1.2 Use your equation of the least squares regression line to estimate Dalvon's pulse rate after 2,5 minutes. Give your answer to the nearest whole number. (2)

2.1.3 The equation of the least squares regression line for Chumani's results is $y = 116{,}79 - 5{,}61x$. The fitter you are, the quicker your pulse rate returns to normal. Who is fitter of Dalvon and Chumani? Give a reason for your answer. (2)

2.2 Three learners cleaned sea birds after an oil spill. They cleaned three different types of birds. The summary of the number of birds they cleaned, as well as the mean and standard deviation of the total number of birds are given in the table below.

	Number of birds by type			**Total number of birds cleaned**	
	Sea gull	**Penguin**	**Cape Cormorant**	**Mean mass in kilograms**	**Standard deviation**
Andrew	2	3	6	1,06	0,42
Bongi	6	2	8	0,77	0,27
Chad	1	0	1	1,50	0

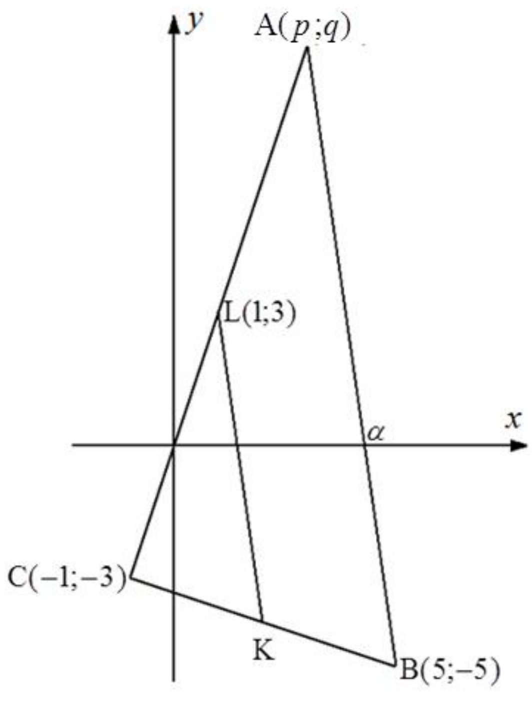
2.2.1 What was the mass of the sea gull that Chad cleaned? Give a reason for your answer. (2)

2.2.2 An award was given to the person who cleaned the greatest mass of birds. Who received the award? (3)

[12]

QUESTION 3

3.1 In the diagram A $(p; q)$, B $(5\,; -5)$ and C $(-1\,; -3)$ are vertices of ΔABC. K is the midpoint of BC and L $(1; 3)$ is the midpoint of AC.



3.1.1 Determine the co-ordinates of K. (2)

3.1.2 Show that $p = 3$ and $q = 9$. (2)

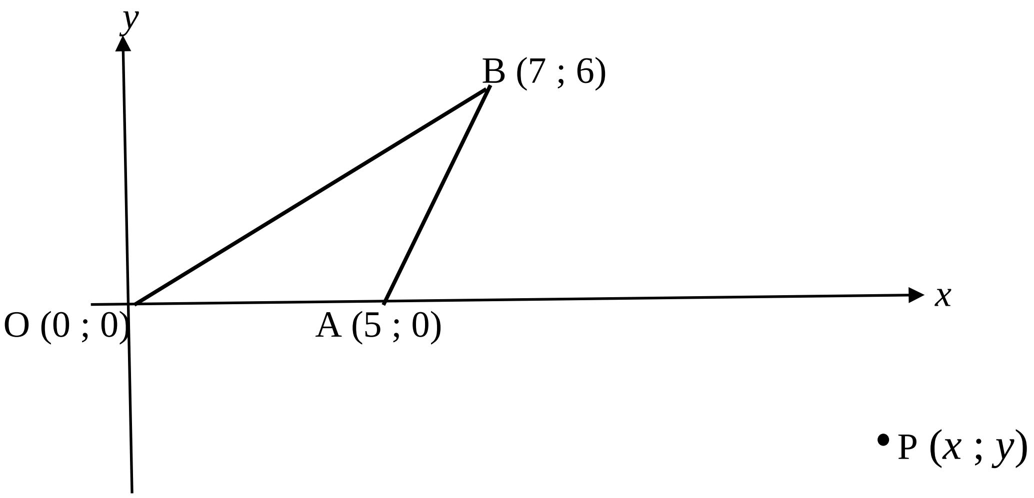
3.1.3 Calculate the size of α. (2)

3.1.4 Calculate the size of $A\hat{B}C$. (4)

3.1.5 Calculate the length of KL. Leave your answer in surd form. (2)

3.1.6 Without any further calculation, write down the length of AB. Give a geometric reason for your answer. (2)

3.2 Three points have co-ordinates $O(0;0)$, $A(5;0)$ and $B(7;6)$. If P is the point $(x;y)$, calculate the values of x and y, given that the area of ΔAOP is 10 units^2. PA = PB and $y < 0$.



(7)

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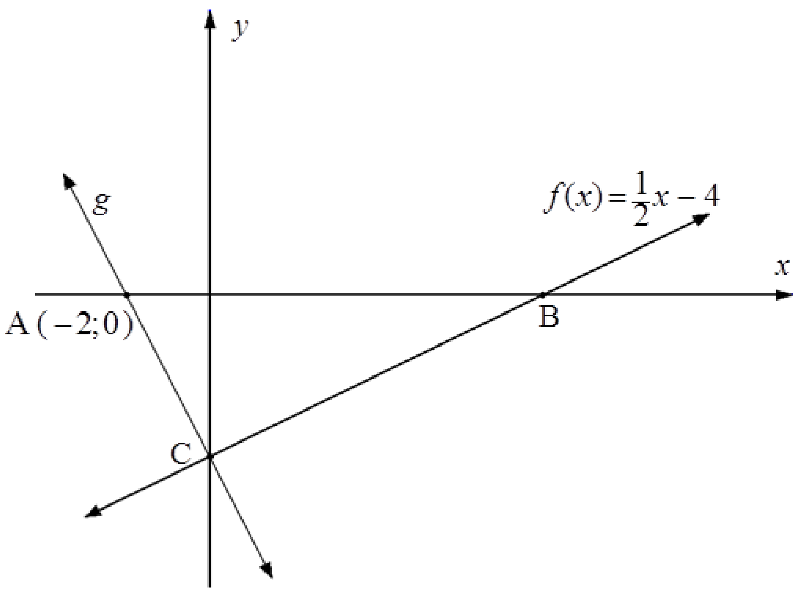
QUESTION 4

4.1 In the diagram the graphs of $f(x) = \frac{1}{2}x - 4$ and g are represented. C is the point of intersection of f and g. C is also the y- intercept of both graphs. A(-2;0) and B are points on the x-axis.



4.1.1 Prove that BC⊥AC (2)

4.1.2 Determine the equation of the circle passing through A, B and C. (4)

4.1.3 Determine the equation of the tangent to the circle through A, B and C, in 4.1.2, at C. (3)

4.2 Determine the value of k if $x^2 + y^2 - 8x + 10y + k = 0$ is the equation of a circle with radius 7 units. (5)

4.3 An ant walks along a path keeping a fixed distance of 3 units from the point $Q(-1;2)$.

4.3.1 Draw a rough sketch on a Cartesian Plane, showing Q and the path the ant follows. (No x- or y- intercepts need to be calculated.) (2)

4.3.2 Write down the equation of the path followed by the ant. (2)

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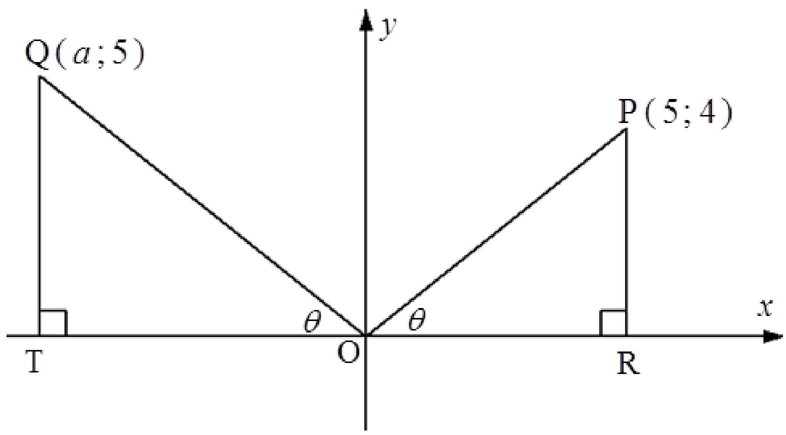
QUESTION 5

CALCULATORS MAY NOT BE USED IN THIS QUESTION.

5.1 In the diagram below similar triangles ΔOPR and ΔOQT are presented. O is the origin. R and T are points on the x-axis.



Determine the following (leave answers in surd form if necessary):

5.1.1 $\sin(90° - \theta)$ (3)

5.1.2 the value of a. (2)

5.2 Given: $\dfrac{\cos(\theta - 180°) \times \cos(\theta + 90°)}{2\tan(180° - \theta)}$

5.2.1 Simplify the given expression to a single trigonometric ratio. (5)

5.2.2 For which values of θ is the expression undefined for $0° \leq \theta \leq 180°$? (3)

5.3 Prove that: $\frac{\sin 2\theta}{\sin\theta} = 4\cos\theta - \frac{\cos 2\theta + 1}{\cos\theta}$ (4)

5.4 Given: $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{5}$

5.4.1 Prove that $\sin A \cos B = 4 \cos A \sin B$ (3)

5.4.2 Hence, calculate the value of $\tan A$ if $\tan B = \frac{1}{2}$. (3)

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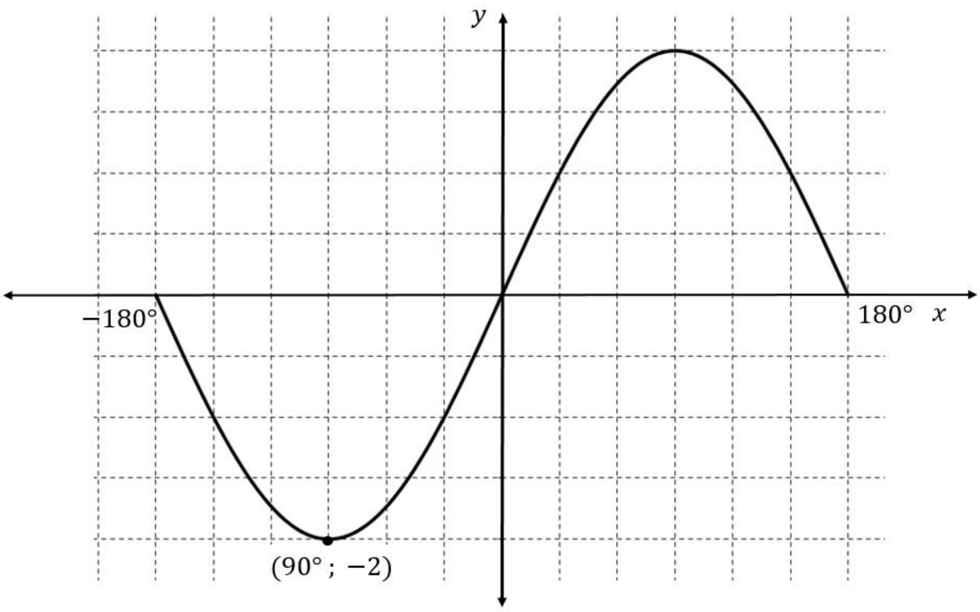
QUESTION 6

In the diagram $g(x) = a\sin x;\ x \in [-180°; 180°]$.



6.1 Determine the value of a. (1)

6.2 Sketch the graph of $f(x) = 2\cos(x - 30°)$ on the same system of axes as provided in the **answer book**. (3)

6.3 Determine the values of x if $f(x) = g(x)$ for $x \in [-180°; 180°]$. (5)

6.4 The minimum value of $y = g(x) - k$ is equal to -5.
Write down the value of k. (1)

6.5 Describe the translation from f to g. (2)

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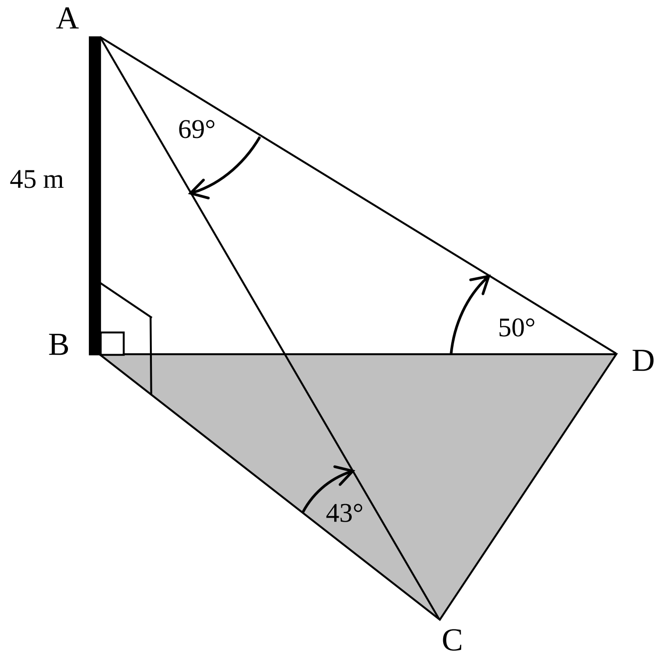
QUESTION 7

In the figure below Thabo is standing at a point A on top of building AB that is 45 m high. He observes two cars at C and D respectively. The cars at C and D are in the same horizontal plane as B. The angle of elevation from C to A is $43°$ and the angle of elevation from D to A is $50°$ and $C\hat{A}D = 69°$.



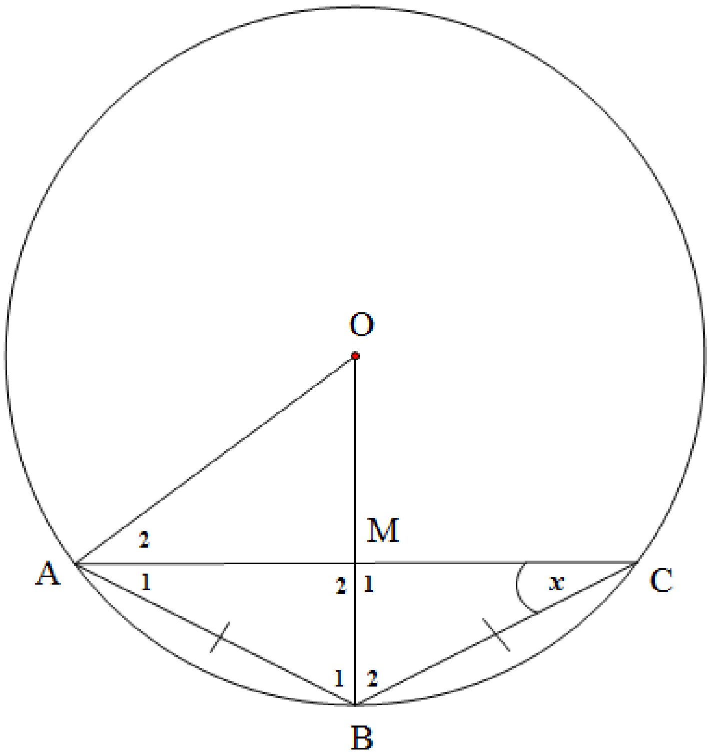
7.1 Calculate the lengths of AC and AD, correct to 2 decimal places. (4)

7.2 Calculate the distance between the two cars, the length of CD. (3)

[7]

QUESTION 8

In the diagram below, a circle passing through A, B and C is drawn. O is the centre of the circle. $AB = BC$ and $A\hat{C}B = x$.



8.1 Give, with reasons, the following angles in terms of x:

8.1.1 $\hat{A}_1$ (1)

8.1.2 $A\hat{O}B$ (2)

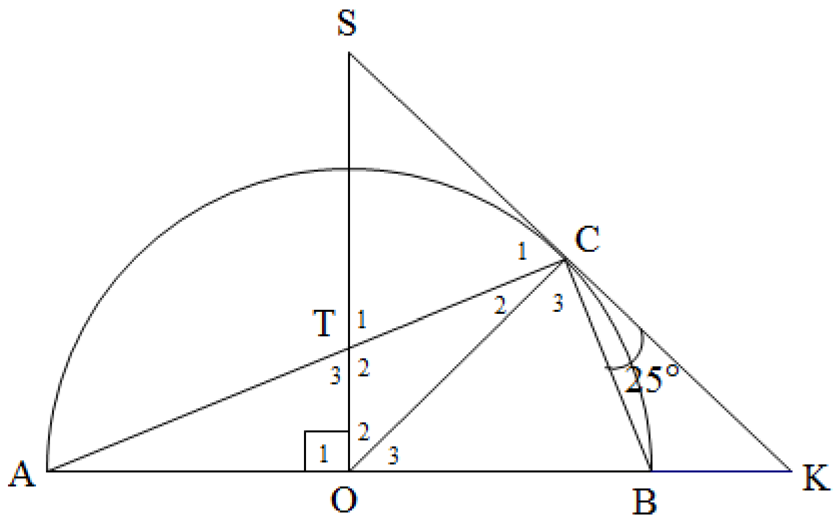
8.2 Prove that $A\hat{M}B = 90°$. (3)

8.3 Determine the length of AM, if it is given that AC = 12 units. (2)

[8]

QUESTION 9

ABC is a semi-circle with centre O. AB is produced to K such that KCS is a tangent to the circle at C. $SO \perp AB$ and $K\hat{C}B = 25^\circ$.



9.1 Give a reason why:

9.1.1 $C\hat{A}B = 25^\circ$ (1)

9.1.2 $O\hat{C}A = 25^\circ$ (1)

9.2 Prove that CTOB is a cyclic quadrilateral. (3)

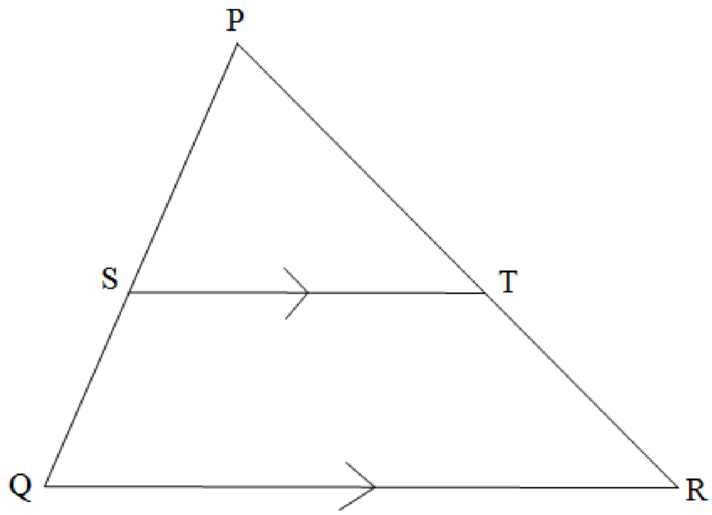
9.3 Write down, with a reason, the size of $C\hat{O}B$. (2)

9.4 Prove that OK is a tangent to the circle passing through S, O and C. (4)

[11]

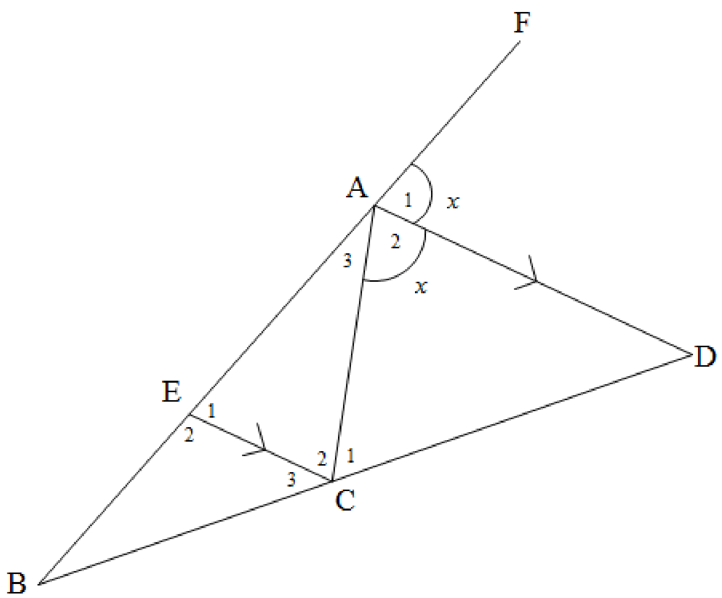
QUESTION 10

10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that ST || QR.



Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

10.2 In ΔABD, BA is produced to F. AC is drawn with C on BD such that $\hat{A}_1 = \hat{A}_2 = x$. CE||DA with E on AB.



10.2.1 Give, with reasons, two more angles equal to x. (3)

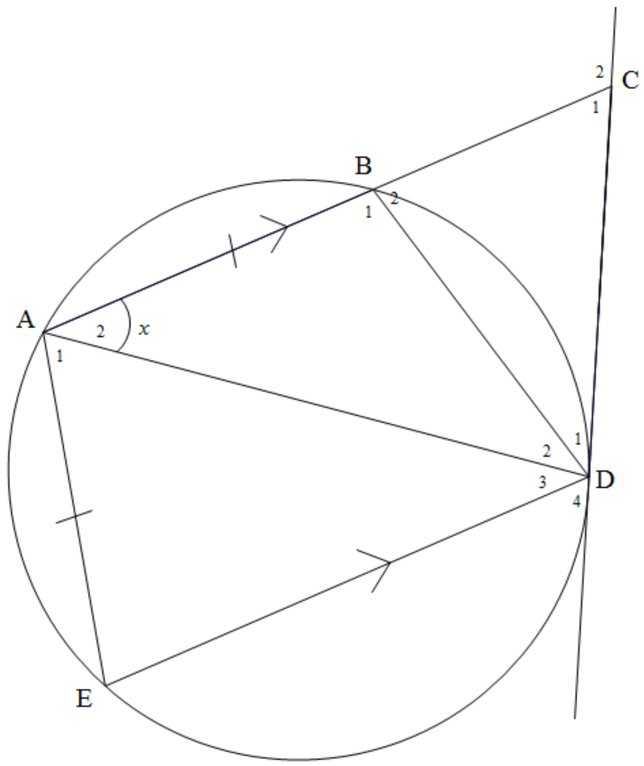
10.2.2 Prove that $\frac{BD}{DC} = \frac{BA}{AC}$. (4)

[13]

QUESTION 11

In the diagram a circle with tangent CD is given. A, B, D and E are points on the circumference of the circle. AE = AB and AB is parallel to ED.

$\hat{A}_2 = x$



11.1 State, with reasons, three more angles equal to x. (6)

11.2 Prove that $\Delta DEA \,|||\, \Delta DBC$ (4)

11.3 Prove that $BC.ED = AE^2$ (5)

11.4 Calculate the value of x , if it is given that $\hat{A}_1 = 75°$. (3)

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TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$