

Background knowledge – Rational numbers

F1 Number systems:

Rational numbers: $\mathbb{Q} = \left\{ \frac{a}{b} / a, b \in \mathbb{Z}, b \neq 0 \right\}$

Remember the properties of 1: (i) $a \times 1 = a$

$$(ii) \quad \frac{a}{1} = a$$

$$(iii) \quad \frac{a}{a} = 1$$

F2 Equivalent fractions:

E.g.1 Write two equivalent fractions for $\frac{1}{2}$.

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6} \quad \text{or} \quad \frac{1 \times 5}{2 \times 5} = \frac{5}{10} \quad [These are \textit{possible} answers!]$$

F3 Arrange rational numbers:

E.g.2 (a) Arrange the following fractions in ascending order: $\frac{1}{2}$; $\frac{3}{4}$ and $\frac{2}{3}$

$$\frac{1}{2} = \frac{6}{12} \quad ; \quad \frac{3}{4} = \frac{9}{12} \quad \text{and} \quad \frac{2}{3} = \frac{8}{12}$$

$$\therefore \frac{1}{2} < \frac{2}{3} < \frac{3}{4}$$

(b) Write a rational number between $\frac{3}{4}$ and $\frac{1}{3}$:

$$\frac{3}{4} = \frac{9}{12} \quad \text{and} \quad \frac{1}{3} = \frac{4}{12}$$

$$\therefore \frac{1}{3} < \frac{5}{12} \text{ or } \frac{6}{12} \text{ or } \frac{7}{12} \text{ or } \frac{8}{12} < \frac{3}{4}$$

F4 Addition and subtraction of fractions:

E.g.3 Simplify:

$$(a) \quad \frac{1}{2} + \frac{4}{3} - \frac{3}{4}$$

$$= \frac{1}{2} \times \frac{6}{6} + \frac{4}{3} \times \frac{4}{4} - \frac{3}{4} \times \frac{3}{3}$$

$$= \frac{6}{12} + \frac{16}{12} - \frac{9}{12}$$

$$= \frac{6 + 16 - 9}{12}$$

$$= \frac{13}{12}$$

$$= \mathbf{1\frac{1}{12}}$$

$$(b) \quad 3x + \frac{2}{3}y - \frac{3}{5}x + 1\frac{3}{4}y$$

$$= \frac{3}{1}x - \frac{3}{5}x + \frac{2}{3}y + \frac{7}{4}y$$

$$= \frac{5}{5} \times \frac{3}{1}x - \frac{3}{5}x + \frac{4}{4} \times \frac{2}{3}y + \frac{3}{3} \times \frac{7}{4}y$$

$$= \frac{15}{5}x - \frac{3}{5}x + \frac{8}{12}y + \frac{21}{12}y$$

$$= \frac{15x - 3x}{5} + \frac{8y + 21y}{12}$$

$$= \frac{12x}{5} + \frac{29y}{12}$$

$$= \mathbf{2\frac{2}{5}x + 2\frac{5}{12}y}$$

E.g.4 Simplify:

$$(a) \quad \frac{2}{3x} + \frac{6}{x}$$

$$= \frac{2}{3x} + \frac{6}{x} \times \frac{3}{3}$$

$$= \frac{2 + 18}{3x}$$

$$= \mathbf{\frac{20}{3x}}$$

$$(b) \quad \frac{y}{2} - \frac{y^2}{5}$$

$$= \frac{y}{2} \times \frac{5}{5} - \frac{y^2}{5} \times \frac{2}{2}$$

$$= \frac{5y}{10} - \frac{2y^2}{10}$$

$$= \mathbf{\frac{5y - 2y^2}{10}}$$

F5 Multiplication and division:

E.g.5 Simplify:

$$(a) \quad \frac{2^1}{3} \times \frac{8}{12^6}$$

$$= \frac{1}{3} \times \frac{8}{12^3}$$

$$= \frac{4}{9}$$

$$(b) \quad 1\frac{1}{3} \div \frac{4}{15}$$

$$= \frac{4}{3} \times \frac{15}{4}$$

$$= \frac{4^1}{3^1} \times \frac{15^5}{4^1}$$

$$= 5$$

E.g.6 Simplify:

$$(a) \quad \frac{4b}{12a} \times \frac{24a}{8b}$$

$$= \frac{4^1 b}{12^1 a} \times \frac{24^2 a}{8^2 b}$$

$$= \frac{2}{2} = 1$$

$$(b) \quad \frac{4y^2}{3} \div \frac{2y}{x}$$

$$= \frac{4^2 y \cdot x}{3} \times \frac{x}{2^1 y}$$

$$= \frac{2xy}{3}$$

F6 Powers and roots:

E.g.7 Calculate, without using a calculator: (a) $\sqrt{\frac{49}{36}}$ (b) $\left(\frac{5-1}{9}\right)^2$

$$(a) \quad \sqrt{\frac{49}{36}} = \frac{\sqrt{49}}{\sqrt{36}} = \frac{7}{6} = 1\frac{1}{6}$$

$$(b) \quad \left(\frac{5-1}{9}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{4^2}{9^2} = \frac{16}{81}$$

F7 Combination of operations:

- Remember the order of operations:
- (1) Brackets
 - (2) Powers and roots
 - (3) Of $\rightarrow \times$
 - (4) Multiplication and division
 - (5) Addition and subtraction

E.g.8 Simplify: $\frac{1}{3} + \frac{2}{3} \div \frac{1}{5} \times \left(\frac{5}{2} - \frac{3}{2}\right)$

$$= \frac{1}{3} + \frac{2}{3} \times \frac{5}{1} \times \frac{\cancel{2}}{\cancel{2}}$$
$$= \frac{1}{3} + \frac{10}{3} = \frac{11}{3} = 3\frac{2}{3}$$

F8 Substitution:

E.g.9 If $a = \frac{1}{2}$; $b = \frac{-1}{3}$ and $c = 4$, then calculate:

(a) abc

$$= \left(\frac{1}{2}\right)\left(\frac{-1}{3}\right)(4)$$

$$= \frac{1}{2^1} \times \frac{-1}{3} \times \frac{4^2}{1}$$

$$= \frac{-2}{3}$$

(b) $ac + b$

$$= \left(\frac{1}{\cancel{2}}\right)\left(\frac{4^2}{1}\right) + \left(\frac{-1}{3}\right)$$

$$= \frac{2}{1} - \frac{1}{3}$$

$$= \frac{2}{1} \times \frac{3}{3} - \frac{1}{3}$$

$$= \frac{6}{3} - \frac{1}{3}$$

$$= \frac{5}{3}$$

$$= 1\frac{2}{3}$$

(c) $2(b + a) + c^2$

$$= 2\left(\frac{-1}{3} + \frac{1}{2}\right) + (4)^2$$

$$= 2\left(\frac{-1}{3} \times \frac{2}{2} + \frac{1}{2} \times \frac{3}{3}\right) + 16$$

$$= 2\left(\frac{-2}{6} + \frac{3}{6}\right) + 16$$

$$= \frac{\cancel{2}}{1}\left(\frac{1}{\cancel{6}^3}\right) + 16$$

$$= \frac{1}{3} + 16$$

$$= 16\frac{1}{3}$$

F9 Calculations with percentages:

- E.g. 10 (a) For a Biology test your marks were 26 out of 40. What is your percentage?
(b) Recently your father earned R5 400 per month. He receives an increase of 12%. Calculate his increased salary per month.
(c) Decrease 80 with 15%.

$$(a) \frac{26}{40} \times \frac{100}{1} = \frac{26}{4^2} \times \frac{10^5}{1} = \frac{26^{13}}{2^1} \times \frac{5}{1} = 13 \times 5 = 65\%$$

$$(b) \text{ Increase} = \frac{5\,400}{1} \times \frac{12}{100} = 54 \times 12 = 648$$

$$\text{Increased salary} = R5\,400 + R648 = R6\,048$$

$$(c) 80 - 15\% \text{ of } 80 = 80 - \frac{15}{100} \times \frac{80}{1} = 80 - \frac{15^3}{10^2} \times \frac{8^4}{1} = 80 - 12 = 68$$

$$(d) \therefore \text{ Compare } 75c \text{ with } 1\,500c \text{ [} R15 = 1\,500c \text{]}$$

$$\therefore \frac{75}{1\,500} \times \frac{100}{1} = \frac{75}{1\,500} \times \frac{100}{1} = \frac{75}{15} = 5\%$$

F10 Order and comparison of fractions:

F10.1 Equivalent fractions:

E.g.11 Write three equivalent fractions for each of the following rational numbers:

$$(a) 0,4$$

$$(b) -3,75$$

$$(a) 0,4 = \frac{4}{10} = \frac{2}{5} = \frac{6}{15}$$

$$(b) -3,75 = -3\frac{75}{100} = -3\frac{3}{4} = \frac{-15}{4}$$

[These are only POSSIBLE answers!]

F10.2 Order of decimal fractions:

Remember: $0,71 > 0,72$; $0,5 > 0,05$; $0,9 > 0,8$ or $-0,4 > -0,5$

F10.3 Rounding off of decimal fractions:

Remember: the symbol $\approx$ means – approximately equal or more or less equal to!

Remember: To round off to the nearest tenth means the same as correct to 1 decimal place.

To round off to the nearest hundredth means the same as correct to 2 decimal places.

To round off to the nearest thousandth means the same as correct to 3 decimal places.

E.g.12 Round off the following, correct to one decimal:

(a) 3,776

(b) -18,045

(c) 5,35

The underlined digit indicates whether the previous digit stays the same or increases by 1.

(a) 3,776
 $\approx 3,8$

(b) -18,045
 $\approx -18,0$

(c) 5,35
 $\approx 5,4$

F11 Operations with decimal fractions:

F11.1 Addition and subtraction of fractions:

E.g.13 Calculate, without using a calculator:

(a) $0,3 + 1,4 = \underline{1,7}$

(b) $5,8 - 5,3 = \underline{0,5}$

E.g.14 Calculate, without using a calculator:

(a) $1,83 + 3,466$

(b) $0,675 - 0,512$

(c) $9 - 3,7$

(a) $\begin{array}{r} 1,83 \\ + 3,466 \\ \hline 5,296 \end{array}$

(b) $\begin{array}{r} 0,675 \\ - 0,512 \\ \hline 0,163 \end{array}$

(c) $\begin{array}{r} 9,0 \\ - 3,7 \\ \hline 5,3 \end{array}$

F11.2 Multiplication and division:

E.g.15 Calculate, without using a calculator:

(a) $38,745 \times 10 = 387,45$

(b) $38,745 \times 100 = 3\,874,5$

(c) $38,745 \times 1\,000 = 38\,745$

(d) $4,23 \times 60 = 4,23 \times 10 \times 6$
 $= 42,3 \times 6$
 $= 253,8$

$\begin{array}{r} 42,3 \\ \times 6 \\ \hline 253,8 \end{array}$

Start multiplication from the back $\therefore$ with 2

E.g.16 Calculate, without using a calculator:

$$(a) \ 9\ 761 \div 10 = \underline{976,1}$$

$$(b) \ 9\ 761 \div 100 = \underline{97,61}$$

$$(c) \ 9\ 761 \div 1\ 000 = \underline{9,761}$$

$$(d) \ 97,61 \div 10 = \underline{9,761}$$

$$\begin{aligned} (e) \ 153,66 \div 60 &= \frac{153,66}{60} \\ &= \frac{153,66}{10 \times 6} \\ &= \frac{15,366}{6} \quad \therefore \text{first divide by 10.} \\ &= \underline{2,561} \end{aligned}$$

$$\begin{array}{r} 2,561 \\ 6 \overline{) 15,3366} \end{array}$$

Start dividing from the front $\therefore$ with 15!

$$(f) \ 841,4 \div 2\ 000 = \frac{841,4}{2\ 000} = \frac{\underline{841},4}{\underline{2\ 000}} = \frac{0,841^14}{2} = \underline{0,4207}$$

F11.3 Powers and roots:

E.g.17 Calculate, without using a calculator:

$$(a) \ (0,2)^3 = 0,2 \times 0,2 \times 0,2 = \underline{0,008} \text{ (Three places after the comma!)}$$

$$(b) \ \sqrt{0,09} = \sqrt{(0,3)^2} = \underline{0,3}$$

F12 Applications:

E.g.8 Thirty friends went to a restaurant. They decided to share the account evenly between them. If the total account was R2 173,80, calculate the amount that each one paid.

$$\text{Each one paid:} \quad R2\ 173,80 \div 30 = \frac{2\ 173,80}{30} = \frac{217,13^18}{3} = 72,46$$

$\therefore$ Each one paid R72,46.