LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

LIMPOPO PROVINCE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2021

MARKS : 150

DURATION : 3 HOURS

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Carefully read the following instructions before answering the questions.

1. This question paper consists of ELEVEN questions.
2. Answer ALL the questions.
3. Number your answers correctly according to the numbering system used in this question paper.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. Show ALL your calculations, diagrams, graphs, etc., that you have used in determining your answers.
6. ANSWERS ONLY will not necessarily be awarded full marks.
7. Where necessary, round off your answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT drawn to scale.
9. Write legibly and present your work neatly.
10. An information sheet with formulae is included at the end of this question paper.

QUESTION 1

1.1 Solve for x :

1.1.1 $x^2 + 4x - 45 = 0$ (3)

1.1.2 $7x^2 - 14x + 3 = 0$ (correct to TWO decimal places) (3)

1.1.3 $(x+3)(2-x) \leq 0$ (3)

1.1.4 $\sqrt{x+34} - x = 4$ (5)

1.1.5 $\dfrac{3^y \text{ x } 9^{x-y}}{27^{x-y}} = 1$ (4)

1.2 Without using a calculator, determine the value of the following expression:

$\left(\sqrt{\sqrt{4}-\sqrt{3}}\right)\left(\sqrt{\sqrt{4}+\sqrt{3}}\right)$ (show all your calculations). (2)

1.3 Solve the following equations simultaneously:

$3x + y - 2 = 0$ and $x^2 + y = xy + x$ (5)

[25]

QUESTION 2

2.1 $2p+1$, $2p-2$ and $4p-3$ are the first three terms of an arithmetic sequence.

2.1.1 Determine the value of p. (2)

2.1.2 If $p = -1$, write down the numerical values of the first three terms. (2)

2.1.3 Calculate the value of T_{50}. (2)

2.2 Consider the quadratic number pattern: 3 ; 11 ; 21 ; 33 ; 47 ; …

2.2.1 The general term of this number pattern is given by

$T_n = an^2 + bn + c$.

Calculate the value of a. (2)

2.2.2 Which term of the sequence equals 497? (6)

[14]

QUESTION 3

3.1 The second term of a geometric series is 24 and its sum to infinity is 100. Determine value(s) of a. (6)

3.2 If it is given that $\sum_{k=1}^{m} 2.2^k < 131068$, determine the highest possible value of m. (5)

[11]

QUESTION 4

Given: $f(x) = \frac{2}{x-4} - 2$

4.1 Write down the equations of the horizontal and vertical asymptotes of f. (2)

4.2 Calculate the x-intercept the intercept of f. (2)

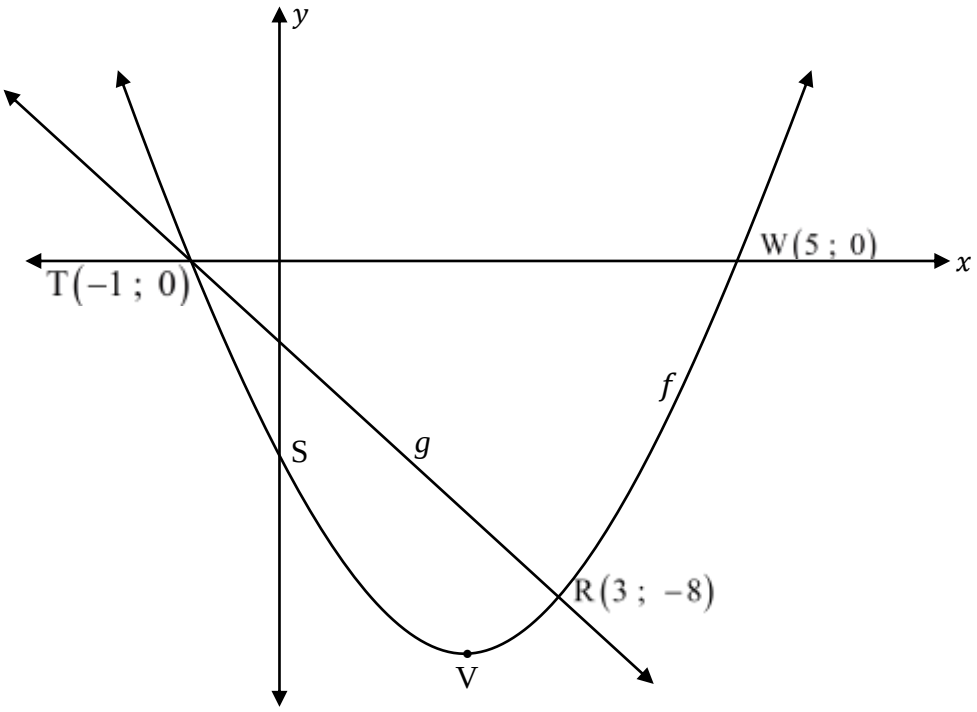
4.3 Sketch the graph of f. Clearly show ALL the intercepts with the axes and the asymptotes. (4)

4.4 The line $y = -x + c$ is the axis of symmetry of f. Determine the value of c. (2)

[10]

QUESTION 5

The sketch graphs of $f(x)=ax^2+bx+c$ and $g(x)=mx+k$ are drawn below. V is the turning point of *f*. The graph f intersects the *x*-axis at T$(-1\ ;\ 0)$ and W$(5\ ;\ 0)$ and the *y*-axis at S. The two graphs intersect at T$(-1\ ;\ 0)$ and R$(3\ ;\ -8)$



5.1 Determine the equation of g in the form of $g(x)=...$ (3)

5.2 Determine the values of a, b and c. (5)

5.3 If $f(x)=x^2-4x-5$, determine,

5.3.1 the turning point V. (3)

5.3.2 the range of *f*. (2)

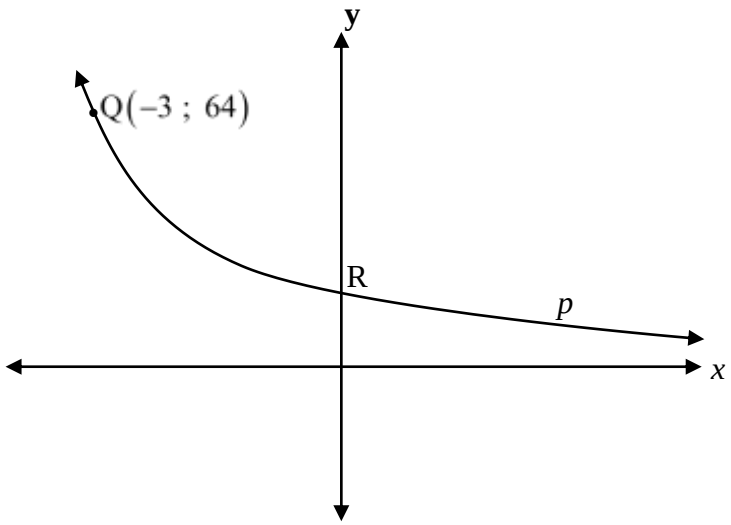
5.4. If $k(x)=f(x-4)+5$, write down the equation of k in the form $k(x)=a(x+p)^2+q$. (3)

5.5 For which values of x is $f'(x).g(x)\geq 0$? (2)

[18]

QUESTION 6

In the diagram below, $Q(-3\ ;\ 64)$ is a point on the graph of $p(x) = b^x$. R is the y-intercept of p.



6.1 Write down the co-ordinates of R. (1)

6.2 Determine the value of b. (3)

6.3 Write down the inverse of p in the form $p^{-1}(x) = \ldots$ (2)

6.4 For which values of x is $0 < \log_{\frac{1}{4}} x < 1$? (2)

[8]

QUESTION 7

7.1 Lawrence bought a car that costs R350 000. The car depreciates annually by 15% p. a., using the reducing balance method. Calculate the book value of the car 5 years after Lawrence bought it. (3)

7.2 Lerato bought a house that costs R800 000. She paid a deposit of R300 000 and the bank granted her a loan to pay for the balance. The bank charges her interest at the rate of 11, 5% p.a., compounded monthly. In order to settle the loan as soon as possible, she makes monthly instalments of R9 500, starting one month after the loan was granted.

7.2.1 How many monthly repayments are required to settle the loan? (4)

7.2.2 Calculate Lerato's final monthly repayment. (5)

7.2.3 Calculate the total interest that Lerato will pay at the end of the first 73 months of the loan period. (2)

[14]

QUESTION 8

8.1 If $f(x) = -2x^2 + 1$, determine $f'(x)$ from the first principle. (5)

8.2 Determine

8.2.1 $\frac{dy}{dx}$ if $y = 2 - x - x^3$ (2)

8.2.2 $D_x\left(-\frac{2x}{\sqrt{x}} - \frac{1}{x}\right)$ (4)

8.2.3 $f'(x)$ if $f(x) = \frac{2x^2 - 16x + 14}{2x - 2}$ (3)

[14]

QUESTION 9

The graph of $f(x) = x^3 - 3x^2 - 9x + 27$ is sketched below. C and E are the turning points of the graph. $B(-2\ ;\ 25)$ is a point on the graph. The graph intersects the *x*-axis at A and E and the *y*-axis at D.

9.1 Determine the coordinates of the maximum turning point of f. (4)

9.2 Determine the equation of the tangent to f at B. (4)

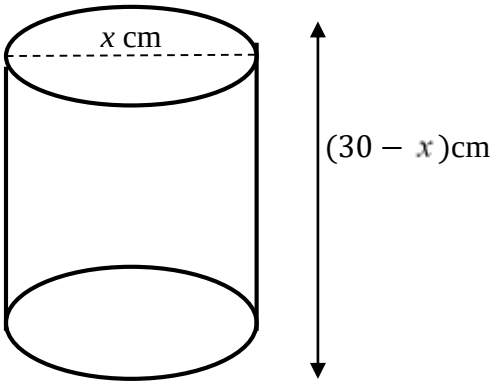
9.3 For which value(s) of k will the equation $x^3 - 3x^2 - 9x + 27 = k$ have exactly one root? (2)

9.4 Determine the value x for which the graph of f' is decreasing. (3)

[13]

QUESTION 10

A cylindrical container without a lid has a diameter of x cm and a height of $(30 - x)$cm.



10.1 Show that the volume of the container is given by $V(x) = \frac{15\pi x^2}{2} - \frac{\pi x^3}{4}$. (2)

10.2 For which value of x will the volume of the container be maximum? (4)

10.3 Calculate the maximum volume of the container.
(correct to TWO decimal places) (2)

[8]

QUESTION 11

11.1 In a certain school there are 60 learners in Grade 10. A survey was conducted among Grade 10 learners taking Mathematics, Life Sciences and Physical Sciences at that school. The survey revealed the following results:

- 30 take Mathematics
- 39 take Life sciences
- 25 take Physical Sciences
- 16 take both Mathematics and Life Sciences
- 10 take both Physical Sciences and Life Sciences
- 4 take Mathematics, Life Sciences and Physical Sciences
- 4 take Mathematics but neither Physical Sciences nor Life Science.

11.1.1 Draw a Venn diagram to represent the information above. (4)

11.1.2 Determine the probability that a learner chosen at random from the Grade10 learners takes either Life Sciences or Physical Sciences but not Mathematics. (2)

11.2 Consider Events A and B with the following conditions:

$P(A) = 0,48$
$P(B) = 0,31$
$P(A \text{ or } B) = 0,67$

Are Events A and B independent? Motivate your answers with relevant calculations. (4)

11.3 Consider the word TRILLION.

11.3.1 Determine the number of different 8-letter words that can be formed using the letters of the word. (2)

11.3.2 Calculate the probability that the vowels will be next to each other in the words formed in 11.3.1. (3)

[15]

TOTAL : 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad T_n = ar^{n-1}$$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ;\ r \neq 1 \qquad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$$

$$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\left(x-a\right)^2 + \left(y-b\right)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$