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PROVINCIAL PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P1

SEPTEMBER 2021

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x.

1.1.1 $x^2 - x - 6 = 0$ (3)

1.1.2 $x(x+6)+1=0$ (correct to TWO decimal places) (4)

1.1.3 $6x - 2x^2 \leq 0$ (3)

1.1.4 $\left(\sqrt{\sqrt{2}-x}\right)\left(\sqrt{\sqrt{2}+x}\right) = x$ (5)

1.2 Solve simultaneously for x and y :

$x - y = 3$ and $x^2 - 3y^2 = 13$ (6)

1.3 If $x^2 = 7$ and $x > 0$, determine the value of x^5 **without using a calculator**. (3)
[24]

QUESTION 2

2.1 The first four terms of a quadratic number pattern are 171; 282; 387; 486 …

2.1.1 Write down the second difference. (2)

2.1.2 Determine the n^{th} term of this pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Another pattern with general term, $P_n = -60n + 2754$, is given. Which term of this new pattern will be the same as that of the quadratic pattern 171; 282; 387; 486 …? (4)

2.2 Consider the geometric series $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$

2.2.1 Calculate the sum of the first 16 terms. (3)

2.2.2 Calculate the value of n such that $S_\infty - T_n = \frac{1023}{4096}$. (5)
[18]

QUESTION 3

Consider: $\sum_{k=1}^{n}(6k+13)$

3.1 Show that $\sum_{k=1}^{n}(6k+13)=3n^2+16n$. (3)

3.2 Hence, calculate the difference between the sum of the first 34 terms and the sum of the first 33 terms. (2)

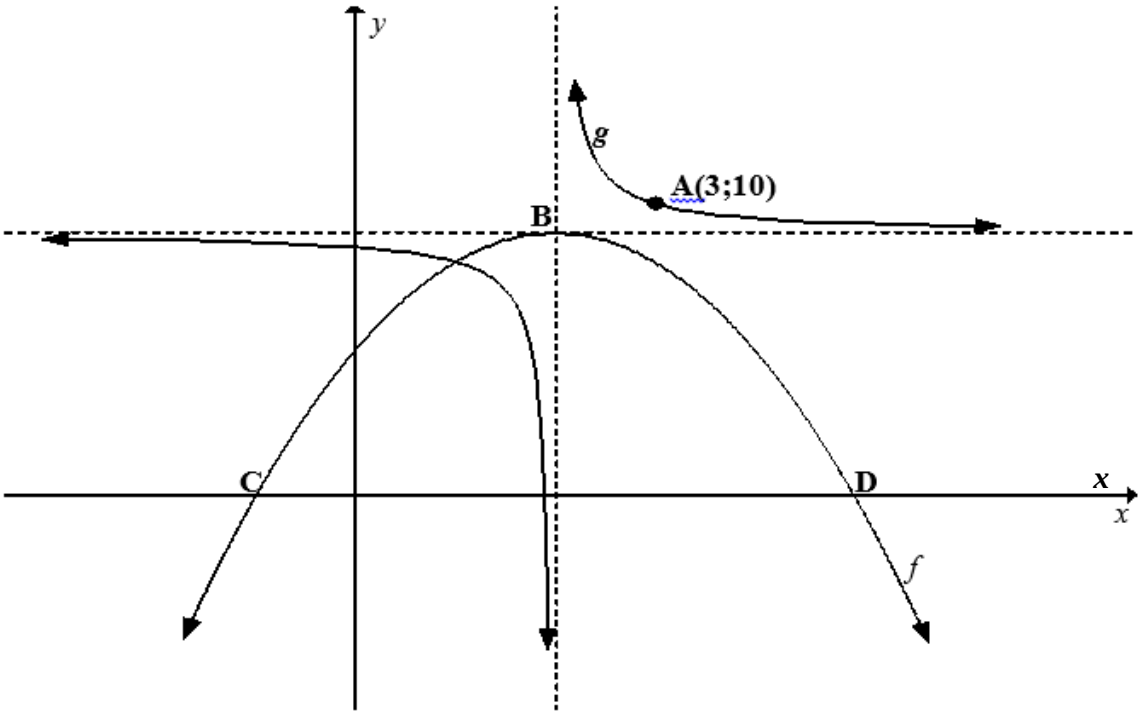
3.3 The first difference of a quadratic number pattern is given by $T_k = 6k+13$. If the fifth term of the quadratic number pattern is 120, determine the value of the third term. (3)

[8]

QUESTION 4

Sketched below are the graphs of $f(x)=-x^2+4x+5$ and $g(x)=\frac{1}{x+p}+q$. B is the turning point of f. The asymptotes of g intersect at B and the point A(3 ; 10) lies on g.

C and D are the x-intercepts of f.



4.1 Determine the coordinates of B. (3)

4.2 Hence, write down the values of p and q. (2)

4.3 Describe the nature of roots of the graph of t, if $t(x)=-f(x)+10$. (2)

4.4 The graph of h, where $h(x)=g(x+m)+n$ has asymptotes $x=4$ and $y=3$. Write down the value(s) of m and n. (2)

4.5 The tangent , $y=8x+k$, touches the graph of f at P. Calculate the coordinates of P. (4)

4.6 Determine the values of x for which:

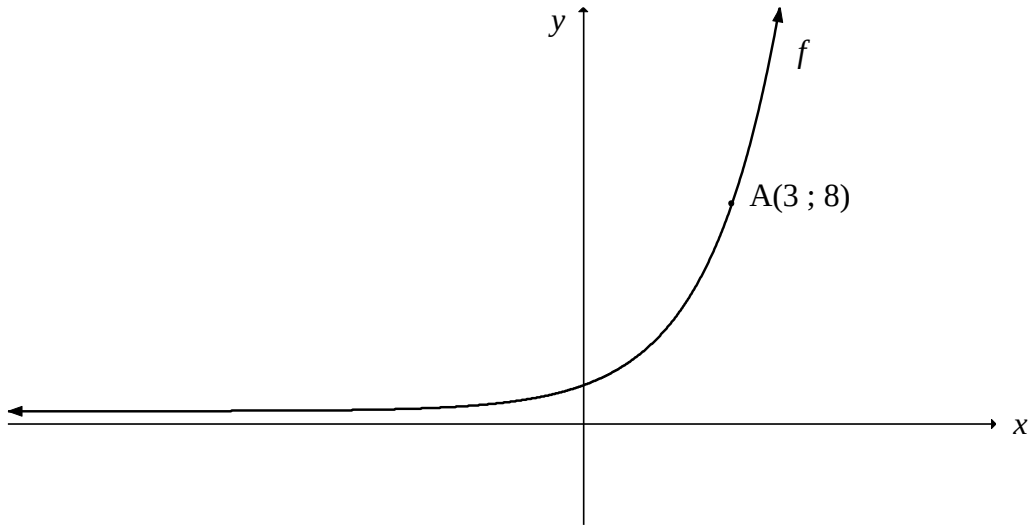
4.6.1 $g(x)\geq 10$ (2)

4.6.2 $f(x).g'(x)>0$ (4)

[19]

QUESTION 5

In the diagram below, the graph of $f(x) = b^x$ is drawn. A(3 ; 8) is a point on *f*.



5.1 Calculate the value of *b*. (2)

5.2 Determine the equation of f^{-1}, the inverse of *f*, in the form $y = ...$ (2)

5.3 Sketch the graph of f^{-1}. Clearly show the intercept(s) with the axes, as well as the coordinates of ONE other point. (3)

5.4 Determine for which values of *x*, will $f^{-1}(x) < 4$. (3)

5.5 Describe the transformation from f to $h(x) = \frac{1}{4} f(x)$. (3)

[13]

QUESTION 6

6.1 Bonolo deposits R4 100 quarterly in a retirement fund for 20 years. The fund earns interest at a rate of 6% p.a. compounded quarterly. The first deposit was made at the end of March of the first year and the last deposit at the end of December of the 20th year.

6.1.1 Calculate the total value of the fund after 20 years? (3)

6.1.2 After 20 years she decides not to withdraw the money but leave it in the fund for another 5 years without making any further payments. The interest rate increases to 6,2% p.a. compounded semi-annually. Calculate the total amount in the fund at the end of the 5-years. (2)

6.2 A couple is planning to purchase their first home. The bank agrees to a loan of R660 000 at an interest rate of 11% p.a. compounded monthly. The loan is repayable over 15 years. The first payment is due at the end of the first month.

6.2.1 Calculate the amount of the monthly instalments. (3)

6.2.2 After the 84th payment, the couple decides to increase their monthly payments. Calculate the balance of the loan after the 84th payment. (3)

6.2.3 From the 85th payment they increase the monthly payments to R10 000. Calculate the number of instalments sooner they will settle the loan. (4)

[15]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x)=4x^2-3$. (5)

7.2 Determine:

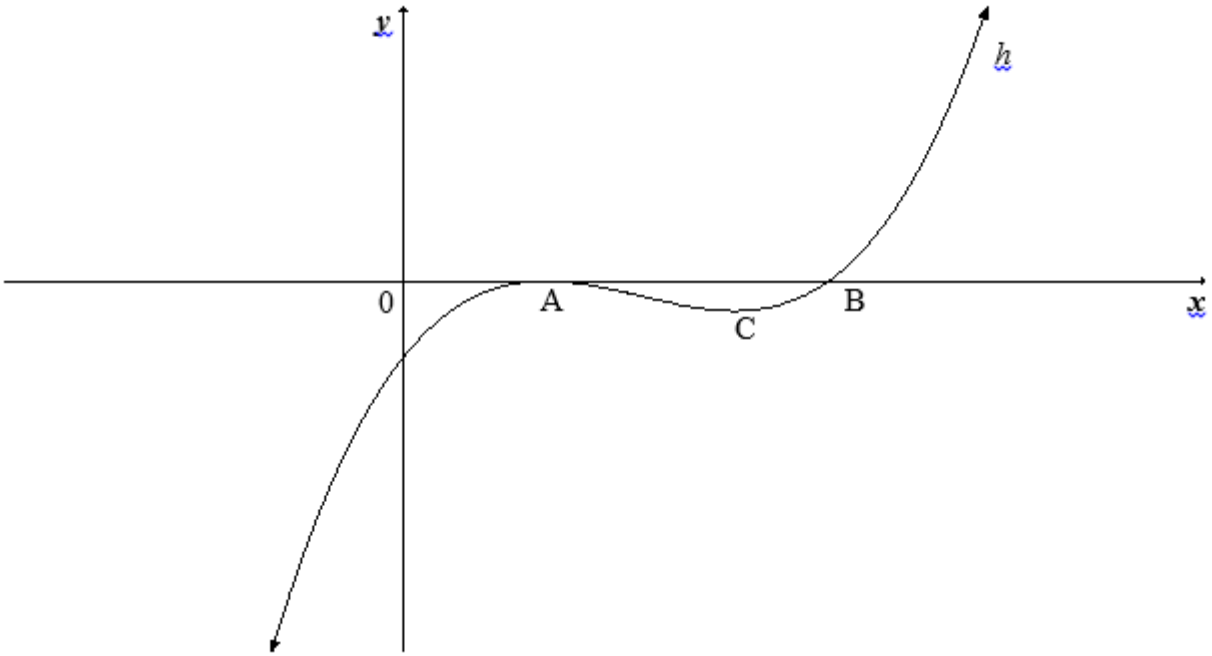
7.2.1 $\frac{dy}{dx}$, if $y=(3x-4)(5x+2)$ (2)

7.2.2 $\frac{d}{dx}\left(x\sqrt{x}-\frac{2}{x^2}\right)$ (4)

[11]

QUESTION 8

In the diagram below, the graph of $h(x)=(x-1)^2(x+k)=x^3-5x^2+7x-3$ is drawn. A and B are the *x*-intercepts and C the minimum turning point of *h*.



8.1 Calculate the coordinates of:

8.1.1 A and B (3)

8.1.2 C (4)

8.2 Determine the value(s) of x for which:

8.2.1 the graph is concave down (3)

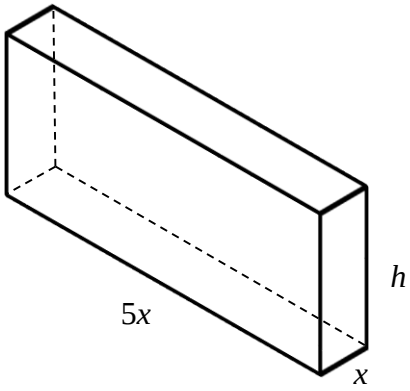
8.2.2 $h(-x)>0$ (2)

8.3 Determine the value(s) of p for which $h(x)+4=p$ has only one solution. (4)

[16]

QUESTION 9

The figure below shows a solid brick in the shape of a rectangular prism. The length is $5x$ units. The width is x units and the height is h units. The total surface area of the brick is 720 cm^2.



9.1 Show that the volume of the brick is given by $V = 300x - \frac{25}{6}x^3$ cubic cm. (5)

9.2 Calculate the maximum volume of the brick. (5)

[10]

QUESTION 10

10.1 Events A and B are given such that $P(A \text{ or } B) = \frac{3}{5}$ and $P(A) = \frac{2}{5}$.

Determine $P(B)$ if:

10.1.1 events A and B are mutually exclusive. (2)

10.1.2 A and B are independent events. (3)

10.2 Events S and T are independent such that $P(S) = P(T) = y$ and $P(S \text{ or } T) = 0{,}84$.
Determine the numerical value of y. (5)
[10]

QUESTION 11

A four digit code is created by using the numerals 0 to 6.

11.1 How many four digit codes can be formed if the numerals can be repeated? (2)

11.2 How many four digit codes can be formed if the code has to be greater than 2 000 but less than 3 000 AND the code must be an even number? A numeral must only be used once. (4)
[6]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$