GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
JUNIE EKSAMEN
GRADE/*GRAAD* 12

2023

MARKING GUIDELINES/
NASIENRIGLYNE

MATHEMATICS/
WISKUNDE

(PAPER/*VRAESTEL* 2)

17 pages/*bladsye*

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aannames van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

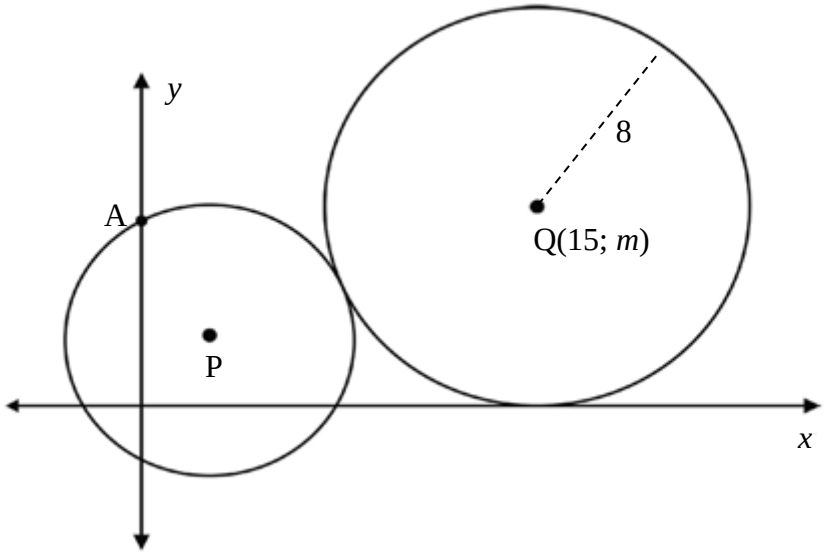
GEOMETRY/*MEETKUNDE*	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	'n Punt vir 'n korrekte bewering *('n Punt vir 'n bewering is onafhanklik van die rede)*
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	'n Punt vir 'n korrekte rede *('n Punt word slegs vir die rede toegeken as die bewering korrek is.)*
S/R	Award a mark if the statement AND reason are both correct.
	Ken 'n punt toe as beide die bewering EN rede korrek is.

QUESTION/*VRAAG* 1

1.1	$m_{AC} = \frac{-3-1}{5+3} = -\frac{1}{2}$	✓ substitution/*substitusie* ✓ answer/*antwoord*	(2)
1.2	The equation of AC will be/*Die vergelyking AC sal wees*: $y-1 = -\frac{1}{2}(x+3)$ $y = -\frac{1}{2}x - \frac{1}{2}$	✓ substitution of gradient/ *substitusie van gradiënt* ✓ substitution of point/ *substitusie van punt* ✓ answer/*antwoord*	(3)
1.3	$\tan\theta = -\frac{1}{2}$ $\theta = 153,43°$	✓ tan… ✓ gradient/*gradiënt* ✓ answer/*antwoord*	(3)
1.4	$m_{BD} = \frac{4}{x-2}$ $m_{AC} = \frac{-4}{8} = -\frac{1}{2}$ BD ⊥ AC given/*is gegee* i.e./*dit is* $\left(\frac{4}{x-2}\right)\times\left(-\frac{1}{2}\right) = -1$ $\frac{4}{x-2} = 2$ $4 = 2x - 4$ $x = 4$ D(4 ; 0)	✓ gradient of BD/ *gradiënt van BD* ✓ gradient of AC/ *gradiënt van AC* ✓ product of gradients = –1/ *produk van gradiënte = –1* ✓ value of *x* /*waarde van x* ✓ coordinates of D/ *koördinate van D*	(5)

1.5	$AD = \sqrt{7^2 + 1^2} = \sqrt{50}$ $AB = \sqrt{(-5)^2 + 5^2} = \sqrt{50}$ $DC = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$ $BC = \sqrt{3^2 + 1^2} = \sqrt{10}$ Two pairs of adjacent sides are equal $\therefore$ ABCD is a kite *Twee paar aangrensende sye is gelyk* $\therefore$ ABCD is 'n vlieër **OR/*OF*** AC $\perp$ BD given/*gegee* DM = MB given/*gegee* One diagonal is bisecting the other at 90°/*Een hoeklyn sny die ander loodrag* $\therefore$ ABCD is a kite/*ABCD is 'n vlieër*	✓ any 2 distances/ *enige 2 afstande* ✓ remaining 2/*oorblywende 2* ✓ reason/*rede* **OR/*OF*** ✓ statement/*bewering* ✓ statement/*bewering* ✓ reason/*rede*	(3)
1.6	$BD = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$ M(3 ; –2) $AM = \sqrt{(-6)^2 + (3)^2} = \sqrt{45} = 3\sqrt{5}$ $\therefore$ Area/Opp. Δ ABD = $\frac{1}{2}$base $\times$ height/ $\frac{1}{2}$*basis* $\times$ *hoogte* $= \frac{1}{2} \times (2\sqrt{5}) \times (3\sqrt{5})$ = 15 sq. units/*vierkante eenhede*	✓ BD ✓ coordinates of M/ *koördinate van M* ✓ AM ✓ substitution into area formula/*substitusie in oppervlak formule* ✓ answer/*antwoord*	(5)
			[21]

QUESTION/*VRAAG* 2



2.1	2.1.1	$(x-15)^2+(y-m)^2=64$	✓answer/*antwoord*	(1)
	2.1.2	(15; 0) lies on c*ircle* Q/*lê op sirkel Q* $(15-15)^2+(0-m)^2=64$ $m=8$ **OR/ *OF*** $m=y_Q=8$	✓ substitution/ *substitusie* ✓ answer/*antwoord* **OR/*OF*** ✓✓ answer only/ *slegs antwoord*	(2)
	2.1.3	$PQ=8+5=13$ **OR/*OF*** $PQ=\sqrt{(3-8)^2+(3-15)^2}$ $=13$ units	✓substitution/ addition/ *substitusie/som van* ✓ answer/*antwoord*	(2)
	2.1.4	The coordinates of/*Die koördinate van* A(0;y) $(0-3)^2+(y-3)^2=25$ $y=7$ A(0;7) **OR/ *OF*** $x_A=0$ $y_A=3+\sqrt{5^2-3^2}=7$ A(0; 7)	✓ substitution/ *substitusie* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde* **OR/*OF*** ✓ substitution/ *substitusie* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde*	(3)

	2.1.5	$m_{AP} = \frac{3-7}{3-0} = \frac{-4}{3}$ $m_{rad} \times m_{\tan} = -1$ $m_{\tan} = \frac{3}{4}$ $y = \frac{3}{4}x + 7$	✓ m_{AP} ✓ $m_{\tan}$ ✓ product of gradients/*voltooiing van vierkant* ✓ equation/*vergelyking*	(4)
2.2	2.2.1	$(x-3)^2 + (y+2)^2 = 12 + 9 + 4$ $(x-3)^2 + (y+2)^2 = 25$ Centre/*middelpunt* is (3; –2) Radius = 5	✓ completing a square/ *voltooiing van vierkant* ✓ equation/*vergelyking* ✓ centre/*middelpunt* ✓ radius	(4)
	2.2.2	KT $= \sqrt{(12-3)^2 + (10+2)^2}$ $= 15$	✓ substitution/*substitusie* ✓ answer/*antwoord*	(2)
	2.2.3	$r_K + r_T = 5 + 10$ $= 15 =$ KT ∴ the circles intersect at **one** point./*die sirkels sny op* ***een plek***	✓ Addition of radii/*som van radiusse* ✓ answer/*antwoord*	(2)
				[20]

QUESTION/*VRAAG* 3

3.1

$$\frac{\sin^2(360° - x) - \cos(x - 180°).\tan(-x).\sin(90° - x).\cos(360° + x)}{\sin(180° + x)}$$

$$= (-\sin x)^2 - \frac{(-\cos x)(-\tan x)(\cos x)(\cos x)}{(-\sin x)}$$

$$= \sin^2 x + \frac{(\cos x)(\frac{\sin x}{\cos x})(\cos^2 x)}{\sin x}$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$

✓ $(-\sin x)^2$
✓ $(-\cos x)$
✓ $(-\tan x)$
✓ $(\cos x)$
✓ $(\cos x)$
✓ $(-\sin x)$
✓ $\sin^2 x + \cos^2 x$
✓ 1

(8)

3.2

$\beta \in [-90°; 270°]$

$$\cos(\beta + 80°) = \frac{\sin(-300°)\cos 45°}{\cos 405°}$$

$$= \frac{(-\sin 300^0)(\cos 45^0)}{\cos 45^0}$$

$$= \sin 60°$$

$$= \frac{\sqrt{3}}{2}$$

$(\beta + 80°) = 30° + k.360° \quad k \in Z$

$\beta = -50° + k.360°$

OR/*OF*

$(\beta + 80°) = -30° + k.360° \quad k \in Z$

$\beta = -110° + k.360°$

$\beta = -50°$ or/*of* $\beta = 250°$

✓ $-\sin 300°$
✓ denominator $\cos 45°$
✓ $\sin 60°$
✓ $\frac{\sqrt{3}}{2}$
✓ $(\beta + 80°) = 30° + k.360°$ $k \in Z$
✓ $\beta = -50°$
✓ $\beta = 250°$

(7)

3.3

3.3.1

$$\frac{\sin 49°}{\cos 41°} = \frac{\sin(90° - 41°)}{\cos 41°}$$

$$= \frac{\cos 41°}{\cos 41°}$$

$$= 1$$

✓ $\frac{\cos 41°}{\cos 41°}$
✓ 1

(2)

3.3.2

$\sin 85° \cos 65° + \cos 85° \sin 65°$

$= \sin(85° + 65°)$

$= \sin 150°$

$= \sin 30°$

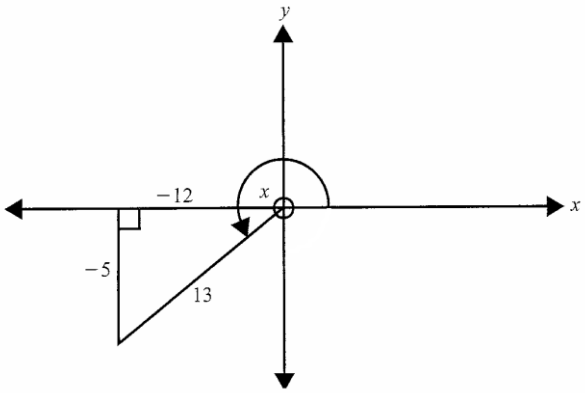
$= \frac{1}{2}$

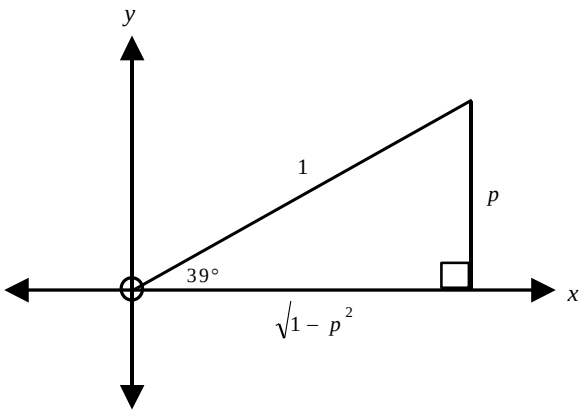
✓ $\sin(85° + 65°)$
✓ $\sin 30°$
✓ $\frac{1}{2}$

(3)

	3.3.3	$\frac{1}{2}(\cos 15° + \sqrt{3}\sin 15°)$ $= \frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ $= \cos 60° \cos 15° + \sin 60° \sin 15°$ $= \cos(60° - 15°)$ $= \cos 45°$ $= \frac{\sqrt{2}}{2}$ **OR/*OF*** $\frac{1}{2}(\cos 15° + \sqrt{3}\sin 15°)$ $= \frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ $= \sin 30° \cos 15° + \cos 30° \sin 15°$ **or/*of*** $= \sin(30° + 15°)$ $= \sin 45°$ $= \frac{\sqrt{2}}{2}$	✓ $\frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ ✓ $\cos 60° \cos 15° + \sin 60° \sin 15°$ ✓ $\cos 45°$ ✓ $\frac{\sqrt{2}}{2}$ ✓ $\frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ ✓ $\sin 30° \cos 15° + \cos 30° \sin 15°$ ✓ $\sin 45°$ ✓ $\frac{\sqrt{2}}{2}$	(4)
				[24]

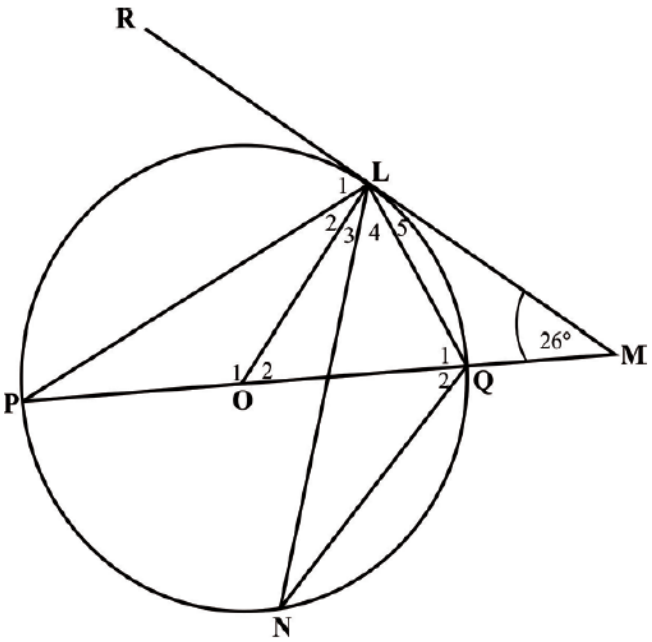
QUESTION/*VRAAG* 4

4.1	$\cos(54° - x) = \sin 2x$ $\cos(54° - x) = \cos(90° - 2x)$ $54° - x = 90° - 2x + k.360° \quad k \in Z$ $x = 36° + k.360°$ or/of $54° - x = -90° + 2x + k.360° \quad k \in Z$ $3x = 144° + k.360°$ $x = 48° + k.120°$	✓ $\cos(54° - x) = \cos(90° - 2x)$ ✓ $54° - x = 90° - 2x + k.360°$. $k \in Z$ ✓ $x = 36° + k.360°$ ✓ $54° - x = -90° + 2x + k.360°.k \in Z$ ✓ $3x = 144° + k.360°$ ✓ $x = 48° + k.120°$	(6)
4.2	$\sin x = -\frac{5}{13}$  $\sin 2x = 2\sin x.\cos x$ $= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$ $= \frac{120}{169}$	✓ diagram ✓ $\sin 2x = 2\sin x.\cos x$ ✓ $\frac{-5}{13}$ ✓ $\frac{-12}{13}$ ✓ $\frac{120}{169}$	(5)
4.3	LHS $= \frac{1 + \sin 2x}{\cos 2x}$ $= \frac{1 + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ $= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ $= \frac{\sin^2 x + 2\sin x\cos x + \cos^2 x}{\cos^2 x - \sin^2 x}$ $= \frac{(\sin x + \cos x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{\cos x + \sin x}{\cos x - \sin x}$ = RHS	✓ $1 + 2\sin x\cos x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $\frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ ✓ $(\sin x + \cos x)^2$ ✓ $(\cos x - \sin x)(\cos x + \sin x)$ ✓ $\frac{\cos x + \sin x}{\cos x - \sin x}$ ✓ = RHS	(7)

4.4	4.4.1	 $\sin 129^0 = \sin 51°$ $= \cos 39°$ $= \sqrt{1-p^2}$	✓ Diagram ✓ $\cos 39°$ ✓ $\sqrt{1-p^2}$	(3)
	4.4.2	$\tan 321° = -\tan 39°$ $= -\frac{p}{\sqrt{1-p^2}}$	✓ $-\tan 39°$ ✓ $-\frac{p}{\sqrt{1-p^2}}$	(2)
	4.4.3	$\sin 78° = 2\sin 39°.\cos 39°$ $= 2p.\sqrt{1-p^2}$	✓ $\sin 2\sin 39°.\cos 39°$ ✓ $2p.\sqrt{1-p^2}$	(2)
				[25]

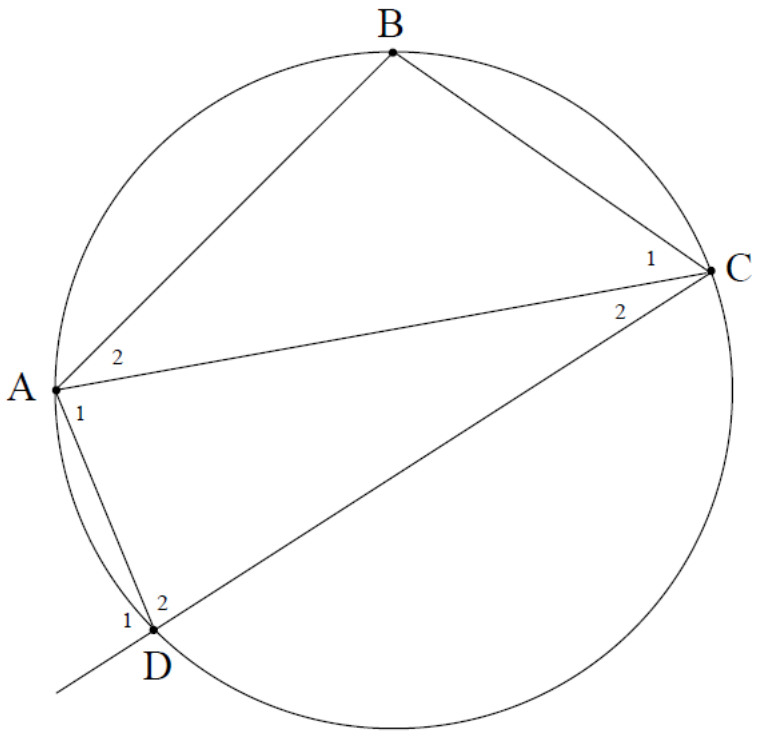
QUESTION/*VRAAG* 5

5.1



5.1.1	$O\hat{L}M = 90°$ (tan ⊥ radius/*raaklyn* ⊥ *radius*)	✓ S ✓ R	(2)
5.1.2	$\hat{O}_2 = 64°$ (sum of ∠s in a triangle/ *som van binnehoeke van Δ*)	✓ S/R	(1)
5.1.3	$\hat{P} = 32°$ (∠ at centre $= 2 \times$ ∠ at circumference)/*middelpuntshoek = 2* x *omtrekshoek*	✓ S ✓ R	(2)
5.1.4	$P\hat{L}Q = 90°$ (∠ in a semi - circle/ ∠ *in 'n halwe sirkel*) $\hat{Q}_1 = 90° - 32° = 58°$ (sum of ∠s in a triangle/ *som van binnehoeke van Δ*) **OR/ *OF*** $\hat{L}_2 = \hat{P} = 32°$ (∠s opp = radii/ ∠*e teenoor = sye/radius se*) $\hat{L}_1 = 58°$ (tan ⊥ radius/ *raaklyn ⊥ radius*) $\hat{Q}_1 = 58°$ (∠s in same seg/ ∠*e in dieselfde seg*)	✓ S ✓ R ✓ S/R **OR** ✓S ✓S ✓ S/R	(3)

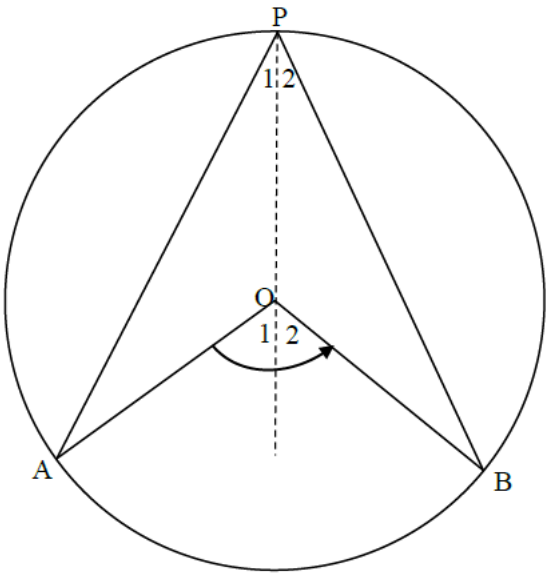
5.2



	$\hat{C}_2 + \hat{A}_1 = \hat{D}_1$ (ext $\angle$ of Δ / *buite* $\angle$ *van* Δ) $\hat{C}_2 = \hat{D}_1 - \hat{A}_1$ $\hat{B} = \hat{D}_1$ (ext $\angle$ of cyclic quad/ *buite* $\angle$ *van kdvh*) $\therefore \hat{C}_2 = \hat{B} - \hat{A}_1$	✓S ✓R ✓S ✓S ✓R	(5)
			[13]

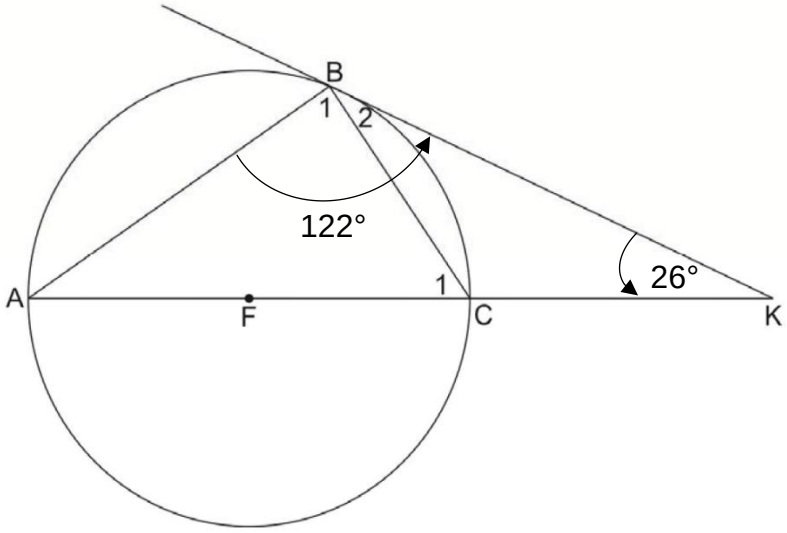
QUESTION/*VRAAG* 6

6.1



	Construction: Draw line PO and extend to create $\hat{O}_1$ *and* $\hat{Q}_2$. *Konstruksie: Trek lyn PO en verleng om* $\hat{O}_1$ *en* $\hat{Q}_2$ *te skep.* OP = OA (radii) $\hat{P}_1 = \hat{A}$ ($\angle$s opp = sides/ *buite* $\angle e$ *van* D) But/ *Maar* $\hat{O}_1 = \hat{P}_1 + \hat{A}$ (ext < of Δ) $\therefore \hat{O}_1 = 2\hat{P}_2$ Similarly/ *Netso* $\hat{O}_2 = 2\hat{P}_2$ $\hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ $\therefore A\hat{O}B = 2A\hat{P}B$	✓ construction/ *konstruksie* ✓ S/R ✓ S/R ✓ S ✓ S	(5)

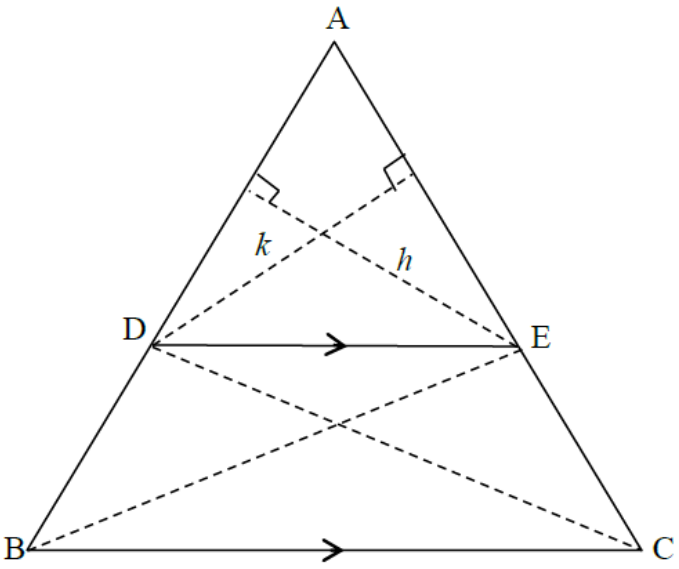
6.2



	$\hat{B}_1 = 90°$ ($\angle$ in a semi-circle) $\therefore\ \hat{B}_2 = 32°$ $A\hat{B}K + \hat{A} + \hat{K} = 180°$ (sum of $\angle$s in Δ) $\therefore\ \hat{A} = 32°$ $\therefore\ \hat{B}_2 = \hat{A} = 32°$ $\therefore$ BK is a tangent (converse : tan-chord theorem)	✓ S/R ✓ S ✓ S ✓ S ✓ R	(5)
			[10]

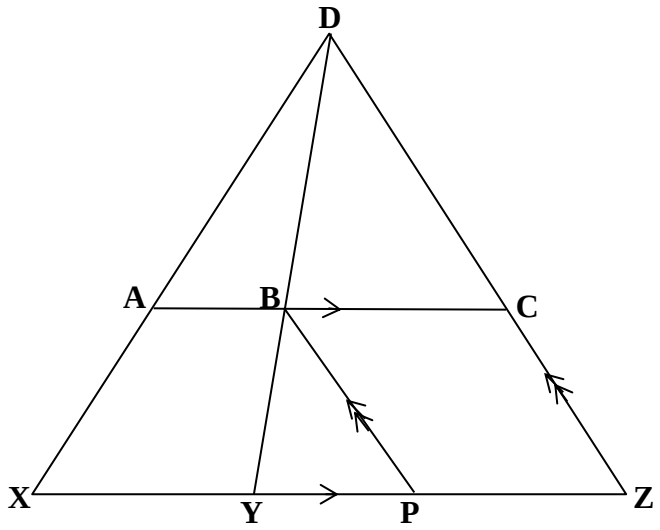
QUESTION/*VRAAG* 7

7.1



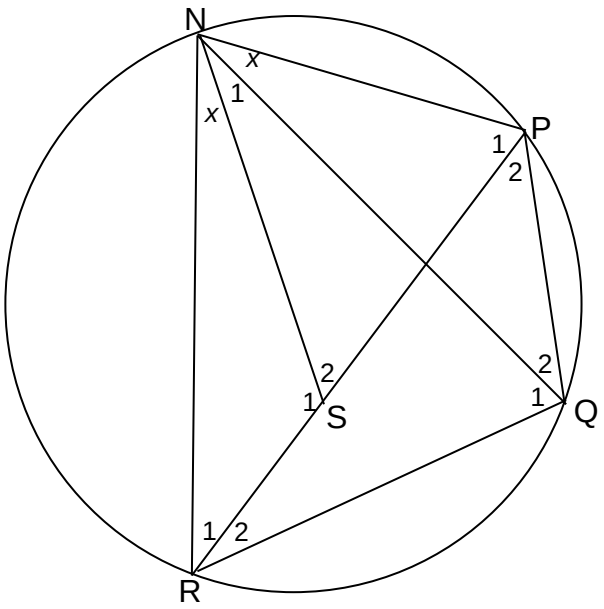
	Construction: In ΔADE, draw height h relative to base AD and height k relative to base AE. Join BE and DC to create ΔBDE and ΔCED. *Konstruksie: In ΔADE, trek hoogte h relatief tot basis AD en hoogte K relatief tot basis AE.* Proof: $\frac{\text{Area of/}\ opp\ van\ \Delta ADE}{\text{Area of/}\ opp\ van\ \Delta BDE} = \frac{\frac{1}{2}AD \times h}{\frac{1}{2}DB \times h} = \frac{AD}{DB}$ $\frac{\text{Area of/}\ opp\ van\ \Delta ADE}{\text{Area of/}\ opp\ van\ \Delta CED} = \frac{\frac{1}{2}AE \times k}{\frac{1}{2}EC \times k} = \frac{AE}{EC}$ But, Area of Δ BDE = Area of Δ CED (same base, same height)/ *maar, opp van Δ BDE = opp van Δ CED (dieselfde basis, dieselfde hoogte)* $\therefore \frac{\text{Area of/}\ opp\ van\ \Delta ADE}{\text{Area of/}\ opp\ van\ \Delta BDE} = \frac{\text{Area of/}\ opp\ van\ \Delta ADE}{\text{Area of/}\ opp\ van\ \Delta CED}$ $\therefore \frac{AE}{EC} = \frac{AD}{DB}$	✓ Construction/ *Konstruksie* ✓ S ✓ S ✓S ✓R ✓S	(6)

7.2



	In ΔDXY: $\frac{DA}{DX} = \frac{DB}{DY}$ (line ‖ to one side of Δ or prop theorem; AB‖XY)/ *(lyn ‖ een sy van Δ of eweredigheidstelling; AB‖XY)* In ΔDYZ: $\frac{ZP}{ZY} = \frac{DB}{DY}$ (line ‖ to one side of Δ or prop theorem; BC‖YZ)/ *(lyn ‖ een sy van Δ of eweredigheidstelling; BC‖YZ)* $\frac{DA}{DX} = \frac{ZP}{ZY}$ ZP = BC (opp. sides of a parm/*oorst sye van parm*) $\frac{BC}{YZ} = \frac{DA}{DX}$	✓S ✓R ✓S ✓S ✓S ✓S/R	(5)
			[11]

QUESTION/*VRAAG* 8



8.1	In ΔNSR and/*en* ΔNPQ $R\hat{N}S = P\hat{N}O$ [given/ *gegee*] $\hat{R}_1 = \hat{Q}_2$ [$\angle^s$ in the same segment/ *omtr* $\angle^e$ *in die sirkel segm*] $\hat{S}_1 = N\hat{P}Q$ [sum of $\angle^s$ in a Δ/*som* $\angle^e$ *van* Δ / $\angle^e$ *van* Δ] $\therefore$ $\Delta NSR \mid\mid\mid \Delta NQP$ [$\angle, \angle, \angle$]	✓ S ✓ S/R ✓ R	(3)
8.2	In ΔNQR and/*en* ΔNPS $R\hat{N}Q = P\hat{N}S$ [$R\hat{N}S = P\hat{N}Q$] $\hat{Q}_1 = \hat{P}_1$ [$\angle^s$ in the same segment]/ *omtr* $\angle^e$ *in die sirkel segm*] $\hat{R} = \hat{S}_2$ [sum of $\angle^s$ in a Δ / *som* $\angle^e$ van Δ] $\therefore$ $\Delta NSR \mid\mid\mid \Delta NPS$ [$\angle, \angle, \angle$]	✓ S ✓ S ✓ R	(3)
			[6]
			TOTAL/*TOTAAL*: 130