MEMO 2023-12ƎD-00007(1) Geometric sequences

(1) Write the next three terms in each of the following sequences:
[Also indicate which of the sequences are geometric sequences. (GS)]

(a) $2\ ;\ 10\ ;\ 50\ ;\ 250\ ;$ **$1\,250\ ;\ 6\,250\ ;\ 31\,250$** **GS**

(b) $31\ ;\ 39\ ;\ 47\ ;\ 55\ ;$ **$63\ ;\ 71\ ;\ 79$**

(c) $1\,011\ ;\ 1\,008\ ;\ 1\,005\ ;\ 1\,002\ ;$ **$999\ ;\ 996\ ;\ 993$**

(d) $-0{,}17\ ;\ -0{,}37\ ;\ -0{,}57\ ;\ -0{,}77\ ;$ **$-0{,}97\ ;\ -1{,}17\ ;\ -1{,}37$**

(e) $\frac{1}{8}\ ;\ \frac{3}{7}\ ;\ \frac{9}{6}\ ;\ \frac{27}{5}\ ;$ **$\frac{81}{4}\ ;\ \frac{243}{3}\ ;\ \frac{729}{2}$**

(f) $\frac{1}{2}\ ;\ 1\ ;\ 2\ ;\ 4\ ;$ **$8\ ;\ 16\ ;\ 32$** **GS**

(2) Write the first four terms in each of the following sequences:
[Also indicate which of the sequences are geometric sequences. (GS)]

(a) $T_n = -4n$ → $T_1 = -4(1) = -4$ $T_2 = -4(2) = -8$

$T_3 = -4(3) = -12$ $T_4 = -4(4) = -16$

∴ $-4\ ;\ -8\ ;\ -12\ ;\ -16$

(b) $T_n = 3^n$ → $T_1 = 3^1 = 3$ $T_2 = 3^2 = 9$

$T_3 = 3^3 = 27$ $T_4 = 3^4 = 81$

∴ $3\ ;\ 9\ ;\ 27\ ;\ 81$ → GS

(c) $T_n = n - 5$ → $T_1 = (1) - 5 = -4$ $T_2 = (2) - 5 = -3$

$T_3 = (3) - 5 = -2$ $T_4 = (4) - 5 = -1$

∴ $-4\ ;\ -3\ ;\ -2\ ;\ -1$

(d) $T_n = 2n + 1$ → $T_1 = 2(1) + 1 = 3$ $T_2 = 2(2) + 1 = 5$

$T_3 = 2(3) + 1 = 7$ $T_4 = 2(4) + 1 = 9$

∴ $3\ ;\ 5\ ;\ 7\ ;\ 9$

(e) $T_n = n^2 - 1$ → $T_1 = (1)^2 - 1 = 0$ $T_2 = (2)^2 - 1 = 3$

$T_3 = (3)^2 - 1 = 8$ $T_4 = (4)^2 - 1 = 15$

∴ $0\ ;\ 3\ ;\ 8\ ;\ 15$

(f) $T_n = 3^{n+1}$ → $T_1 = 3^{1+1} = 3^2 = 9$ $T_2 = 3^{2+1} = 3^3 = 27$

$T_3 = 3^{3+1} = 3^4 = 81$ $T_4 = 3^{4+1} = 3^5 = 243$

∴ $9\ ;\ 27\ ;\ 81\ ;\ 243$ → GS

(3) Consider the sequence: $6\ ;\ 2\ ;\ \frac{2}{3}\ ;\ \frac{2}{9}\ ;\ \ldots\ldots$

(a) Calculate the general term of the sequence.

$a = 6$

$r = \frac{1}{3}$

$n = \ ?$

$T_n = ?$

$T_n = a.r^{n-1}$

$\therefore \quad T_n = 6.\left(\frac{1}{3}\right)^{n-1}$

or

$\therefore \quad T_n = 6(3^{-1})^{n-1}$

$\therefore \quad T_n = 6(3)^{1-n}$

(b) Calculate T_{11}.

$a = 6$

$r = \frac{1}{3}$

$n = 11$

$T_{11} = \ ?$

$T_n = 6.(3)^{1-n}$

$\therefore \quad T_{11} = 6 \times 3^{1-11}$

$\therefore \quad T_{11} = 6 \times 3^{-10}$

$\therefore \quad T_{11} = \frac{2}{19\,683}$

(c) Calculate n if $T_n = \frac{2}{729}$.

$T_n = 6.\left(\frac{1}{3}\right)^{n-1}$

$\therefore \quad \frac{2}{729} = 6.\left(\frac{1}{3}\right)^{n-1}$

$\therefore \quad \frac{2}{729 \times 6} = \left(\frac{1}{3}\right)^{n-1}$

$\therefore \quad \frac{1}{2\,187} = \left(\frac{1}{3}\right)^{n-1}$

$\therefore \quad \left(\frac{1}{3}\right)^{7} = \left(\frac{1}{3}\right)^{n-1} \quad \rightarrow \quad 3^7 = 2\,187$

$\therefore \quad 7 = n - 1 \quad \rightarrow \quad EB \Leftrightarrow EE$

$\therefore \quad n = 8$

(4) Consider the sequence: $-5\ ;\ 10\ ;\ -20\ ;\ 40\ ;\ \ldots\ldots$

(a) Is this sequence converging? Motivate.

$r = \frac{T_2}{T_1} = \frac{10}{-5} = -2$

$\therefore$ The sequence is not converging, because $r < -1$.

(b) Calculate the general term of the sequence.

$a = -5$

$r = -2$

$T_n = ?$

$T_n = a.r^{n-1}$

$\therefore \quad T_n = -5.(-2)^{n-1}$

(c) Calculate T_{16}.

$T_n = -5.(-2)^{n-1}$

$\therefore \quad T_{16} = -5.(-2)^{16-1}$

$\therefore \quad T_{16} = -5.(-2)^{15}$

$\therefore \quad T_{16} = 163\,840$

(d) Calculate n if $T_n = -5\,120$.

$$T_n = -5 \cdot (-2)^{n-1}$$

$$\therefore \quad -5\,120 = -5 \cdot (-2)^{n-1}$$

$$\therefore \quad \frac{-5\,120}{-5} = -5 \cdot (-2)^{n-1}$$

$$\therefore \quad 1\,024 = (-2)^{n-1}$$

$$\therefore \quad (-2)^{10} = (-2)^{n-1}$$

$$\therefore \quad 10 = n - 1 \qquad \rightarrow \qquad \text{EB} \Leftrightarrow \text{EE}$$

$$\therefore \quad n = 11$$