



LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE/GRAAD 12

**MATHEMATICS PAPER 2/ WISKUNDE V2
SEPTEMBER 2020
MEMORANDUM**

MARKS/PUNTE: 150

This memorandum consists of 15 pages / Hierdie memo bestaan uit 15 bladsye.

QUESTION /VRAAG 1

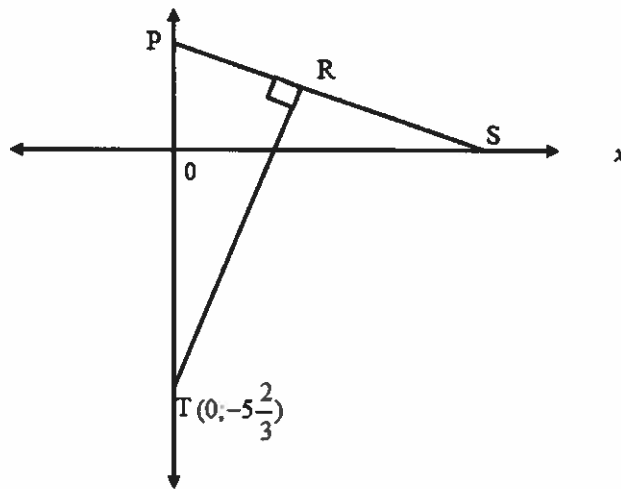
1.1.1		✓box /mond ✓whiskers, on scale/ stert, en op skaal (2)
1.1.2	School B/skool B: In B 50% of learners got above 60% whilst 50% of learners of A above 55% / 50% van B het meer as 60% ; by A het 50% meer as 55% median of B > median of A/ mediaan van B > mediaan van A 75% of B above 30% and at A 75% above 23,5% / 75% van B bo 30% ; By A 75% bo 23,5% OR/OF Skewed to the right/ skeef na Regs	✓School /skool B ✓✓any two reasons enige twee redes ✓✓ OR/ OF ✓✓ (3)
1.2	$\frac{135 \times 24 + 225 \times 32 + 200 \times x}{560} = x$ $3240 + 7200 + 200x = 560x$ $360x = 10440$ $x = 29$	✓numerator/ teller ✓560 ✓360x = 10440 ✓ answer/antwoord (4)
		[9]

QUESTION/VRAAG 2

2.1	$a = 23,19$ $b = 0,768..$ $y = 23,19 + 0,77x$	✓a ✓b ✓ $y = 23,19 + 0,77x$ (3)
2.2	$y = 23,19 + 0,77(30)$ $= 46,29$ $\approx 46\%$	✓ substitution/vervanging ✓answer/antwoord (2)

2.3.1	$\delta = 10,99$	✓✓ answer /antwoord (2)
2.3.2	$57,4\% - 49,4\% = 8\%$ $\frac{x}{50} = \frac{8}{100}$ $x = 4$ marks	✓8% ✓4 marks/ punte (2)
		[9]

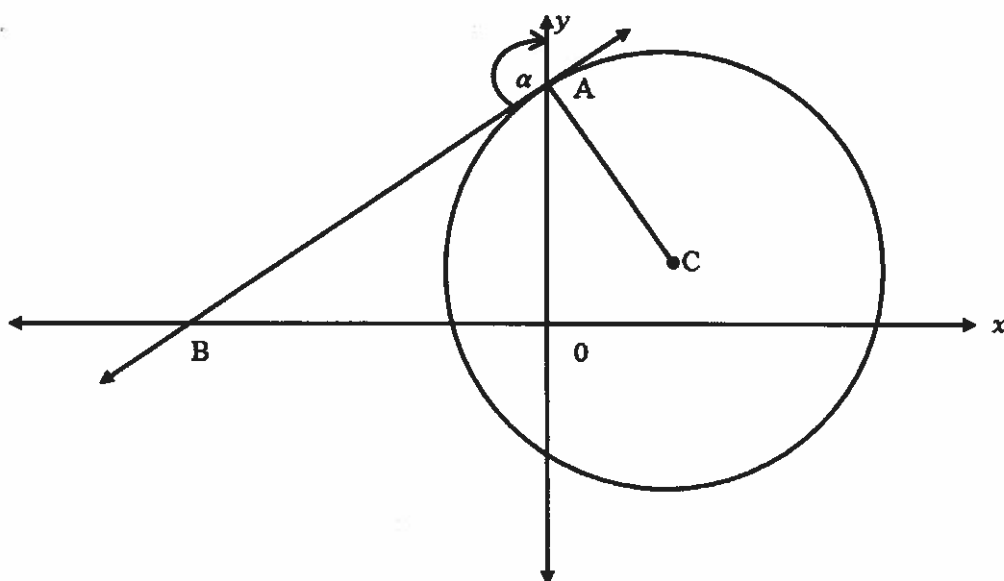
QUESTION /VRAAG 3



3.1	$x + ay = a$ $0 + ay = a$ $y = 1$ $\therefore P(0;1)$	✓ $x=0$ ✓ $y=1$ ✓ $P(0;1)$ (3)
3.2	$OS : OP = 3 : 1$ $S(3;0) : x + ay = a$ $3 + a(0) = a$ $a = 3$	✓ $S(3;0)$ ✓substitution/vervang ✓ $a = 3$ (3)
3.3	$m_{PS} = -\frac{1}{3}$ $m_{TR} = 3$ RT: $y = 3x - 5\frac{2}{3}$	✓ $m_{PS} = -\frac{1}{3}$ ✓ $m_{TR} = 3$ ✓equation/ vergelyking (3)
3.4	$x + 3y = 3 \Rightarrow y = -\frac{1}{3}x + 1$ $\therefore -\frac{1}{3}x + 1 = 3x - \frac{17}{3}$ $-x + 3 = 9x - 17$ $20 = 10x$	✓equating /stel gelyk

	Substitute: $y = -\frac{1}{3}(2) + 1$ $y = \frac{1}{3}$ $R(2; \frac{1}{3})$	✓ simplification/ <i>vereenvoudig</i> ✓ $x = 2$ ✓ $y = \frac{1}{3}$ (4)
3.5	$area\Delta PRT = \frac{1}{2} \times PT \times h$ $= \frac{1}{2} \times 6\frac{2}{3} \times 2$ $= \frac{20}{3}$ sqd units/ <i>vierkante eenhede</i>	✓ $PT = 6\frac{2}{3}$ ✓ $h = 2$ ✓ answer/ <i>antwoord</i> (3)
3.6	PT is the diameter(converse: $\angle$ in semi $\odot$)/ $\odot PT$ is die middellyn (omgekeerde: $\angle$ in semi $\odot$) $radius = \frac{10}{3}$ units.	✓ S/R ✓ answer / <i>antwoord</i> (2)
		[18]

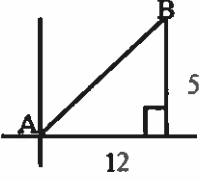
QUESTION/VRAAG 4



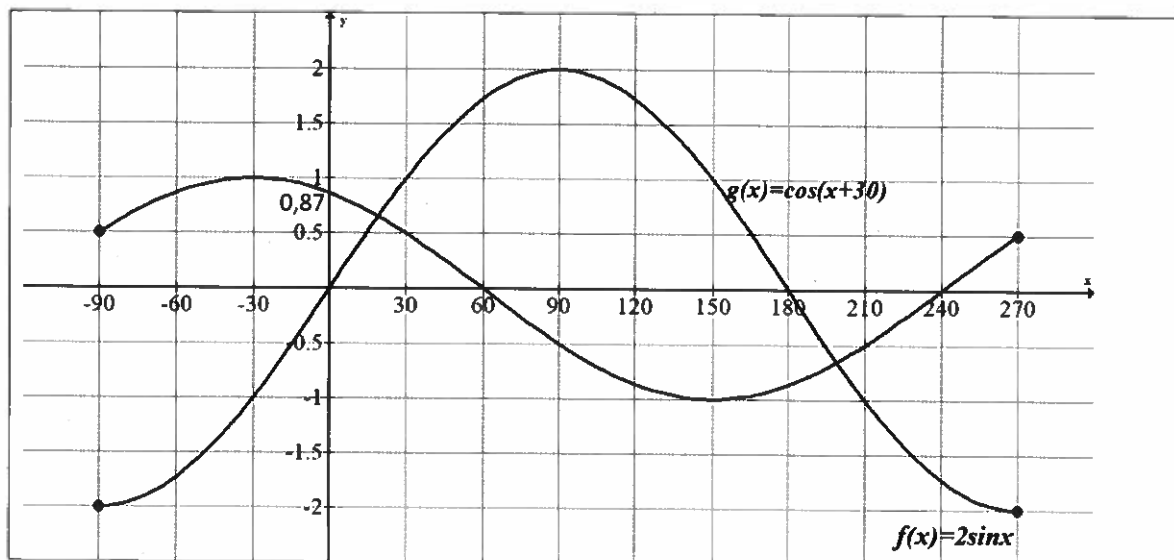
4.1.1	$x^2 - 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4$ $(x - 3)^2 + (y - 2)^2 = 25$ $C(3;2)$ $r = 5$	Completing the square <i>vierkantsvoltooiing</i> ✓LHS/ LK ✓RHS/RK ✓ C(3;2) ✓ r = 5 (4)
4.1.2	Y-intercept: /Y as: $x = 0$ $y^2 - 4y - 12 = 0$ $(y - 6)(y + 2) = 0$ $y = -2 \text{ or } y = 6 \therefore A(0;6)$ $m_{AC} = \frac{6 - 2}{0 - 3} = -\frac{4}{3}$ $m_{AB} = \frac{3}{4} \text{ (tan } \perp \text{ radius)}$ $y = \frac{3}{4}x + 6$	✓ $x = 0$ ✓ standardform/ <i>standaardvorm</i> ✓ factors/faktore ✓ A(0;6) ✓ $m_{AC} = -\frac{4}{3}$ ✓ $m_{AB} = \frac{3}{4}$ ✓ R ✓ equation/ <i>vergelyking</i> (8)
4.1.3	Let $A = \theta$: $\tan \theta = \frac{3}{4}$ $\theta = 36,87^\circ$ $\alpha = 90^\circ + 36,87^\circ$ $\alpha = 126,87^\circ$	✓ $\tan \theta = \frac{3}{4}$ ✓ $\theta = 36,87^\circ$ ✓ $\alpha = 90^\circ + 36,87^\circ$ ✓ answer / <i>antwoord</i> (1)

4.2	$x^2 + 2x + 1 + y^2 - 4y + 4 = 44 + 1 + 4$ $(x+1)^2 + (y-2)^2 = 49$ Centre / <i>middelpunt</i> D(-1;2) ; radius = 7 $CD = 3 - (-1) = 4$ $r_C + r_D = 5 + 7 = 12 > 4$ $\therefore$ the two circles intercept internally/ <i>twee sirkels sny inwendig</i>	✓Centre D(-1;2) ✓ $r = 7$ ✓ $CD = 4$ ✓ $r_C + r_D = 12$ ✓ > 4 ✓conclusion / <i>gevolgtrekking</i> (6)
		[22]

QUESTION/VRAAG 5

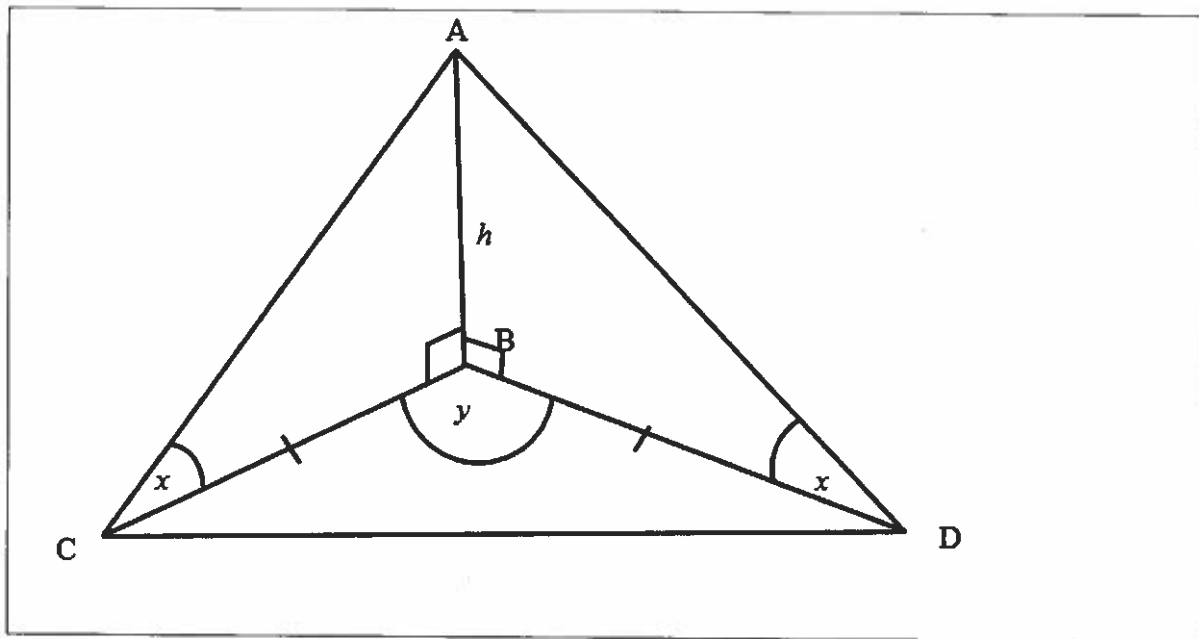
5.1	$\tan A = \frac{5}{12}$  $r^2 = 12^2 + 5^2$ $= 169$ $r = 13$ $13 \sin A - 3 \tan B$ $= 13\left(\frac{5}{13}\right) - 3\left(\frac{12}{5}\right)$ $= 5 - \frac{36}{5}$ $= -\frac{11}{5}$	✓diagram ✓ $r = 13$ ✓ $\sin A = \frac{5}{13}$ ✓ $\tan B = \frac{12}{5}$ ✓answer / <i>antwoord</i> (5)
5.2	$\frac{-\tan 60^\circ - \sin x}{\sin x + 2 \cos 30^\circ}$ $= \frac{-\sqrt{3} - \sin x}{\sin x + 2\left(\frac{\sqrt{3}}{2}\right)}$ $= \frac{-(\sin x + \sqrt{3})}{(\sin x + \sqrt{3})}$ $= -1$	✓ $-\tan 60^\circ$ ✓ $-\sin x$ ✓ $2 \cos 30^\circ$ ✓special angles/ <i>bekende hoeke</i> ✓factorisation/ <i>faktoriseer</i> ✓answer/

5.3.1	$\begin{aligned}\cos(x+y) &= \sin(90^\circ - (x+y)) \\ &= \sin((90^\circ - x) + (-y)) \\ &= \sin(90^\circ - x)\cos(-y) + \cos(90^\circ - x)\sin(-y) \\ &= \cos x \cos y - \sin x \sin y\end{aligned}$	✓Correct co-fn/ korrekte ko – funksie ✓ regrouping/ herrangskik ✓Correct expansion/ uitbreiding ✓simplification/ vereenvoudig (4)
5.3.2	$\begin{aligned}\cos(x-y) - \cos(x+y) &= \cos x \cos y + \sin x \sin y - (\cos x \cos y - \sin x \sin y) \\ &= \cos x \cos y + \sin x \sin y - \cos x \cos y + \sin x \sin y \\ &= 2 \sin x \sin y\end{aligned}$	✓Expansion/ brei uit ✓simpilification / vereenvoudig (2)
5.3.3	$\begin{aligned}2 \sin 195^\circ \sin 45^\circ &= \cos(195^\circ - 45^\circ) - \cos(195^\circ + 45^\circ) \\ &= \cos 150^\circ - \cos 240^\circ \\ &= -\cos 30^\circ - (-\cos 60^\circ) \\ &= -\frac{\sqrt{3}}{2} + \frac{1}{2} \\ &= \frac{-\sqrt{3} + 1}{2}\end{aligned}$	✓Substitution/ vervang ✓ $-\cos 30^\circ$ ✓ $-\cos 60^\circ$ ✓Special angles/ bekende hoeke ✓answer/antwoord (5)
		[22]

QUESTION/VRAAG 6

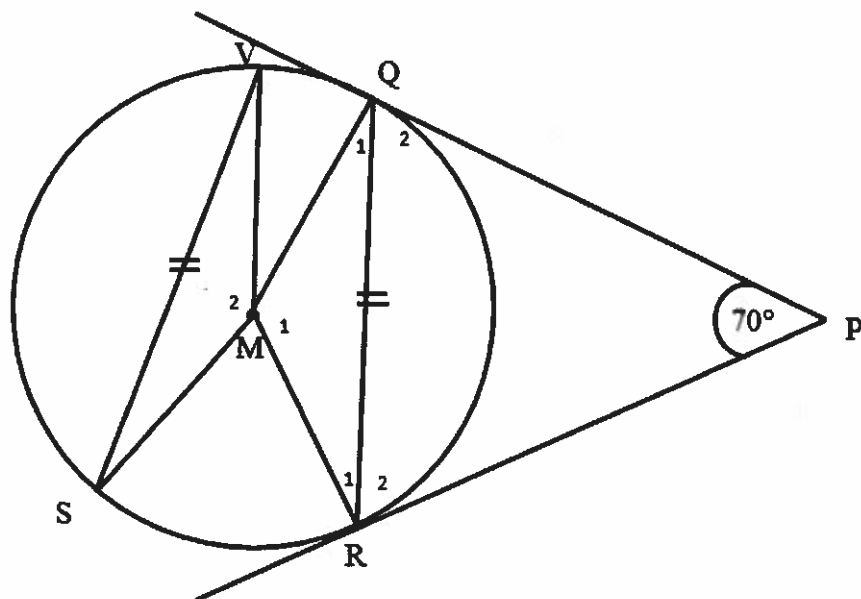
6.1	g : ✓ x-intercepts/ x – asse ✓ y-intercepts/y – as ✓ turningpoints / draaipunte		(3)
6.2	$2\sin x = \cos x \cos 30^\circ - \sin x \sin 30^\circ$ $2\sin x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$ $\frac{5}{2} \sin x = \frac{\sqrt{3}}{2} \cos x$ $\frac{\sin x}{\cos x} = \frac{\sqrt{3}}{5}$ $\tan x = \frac{\sqrt{3}}{5}$	✓ Expansion of g / <i>brei uit</i> ✓ Special angles/ <i>bekende hoeke</i> $\checkmark \frac{5}{2} \sin x = \frac{\sqrt{3}}{2} \cos x$ $\checkmark \frac{\sin x}{\cos x}$	(4)
6.3	$\tan x = \frac{\sqrt{3}}{5}$ $x = 19,1^\circ$ or $x = 180^\circ + 19,1^\circ = 199,1^\circ$	✓ $19,1^\circ$ ✓ $199,1^\circ$	(2)
6.4.1	$x \in [60^\circ; 180^\circ] \cup [240^\circ; 270^\circ]$	✓ 60° and / en 180° ✓ 240° and / en 270° ✓ notation / notasie	(3)
6.4.2	$x \in [19,1^\circ; 199,1^\circ]$	✓ critical values / kritieke waardes ✓ notation / notasie	(2)

QUESTION /VRAAG 7



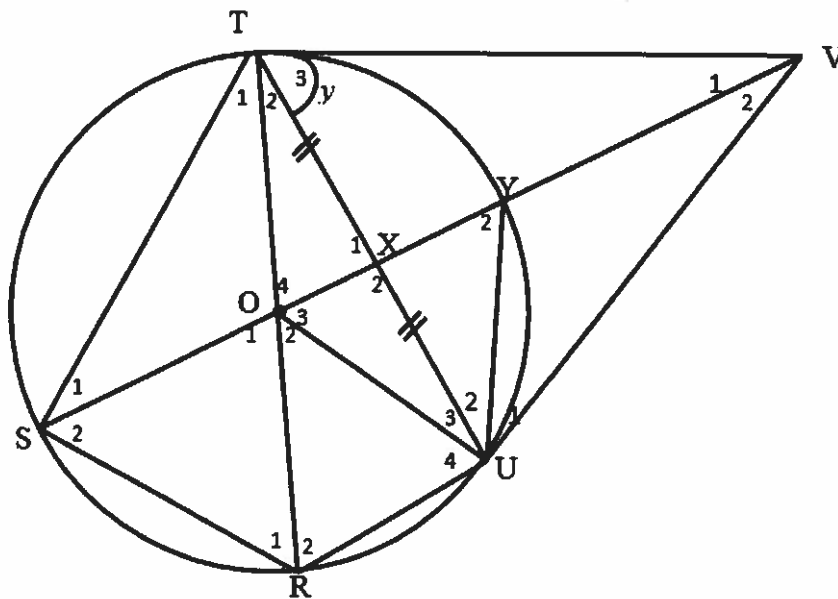
$\text{In } \triangle ABC : \tan x = \frac{h}{CB}$ $CB = \frac{h}{\tan x}$ $\text{In } \triangle BCD : CB = BD$ $CD^2 = BC^2 + BD^2 - 2BC \times BD \times \cos y$ $=$ $\left(\frac{h}{\tan x}\right)^2 + \left(\frac{h}{\tan x}\right)^2 - 2 \frac{h}{\tan x} \times \frac{h}{\tan x} \times \cos y$ $= \frac{2h^2}{\tan^2 x} - \frac{2h^2 \cos y}{\tan^2 x}$ $= \frac{2h^2}{\tan^2 x} (1 - \cos y)$ $CD = \frac{h}{\tan x} \sqrt{2(1 - \cos y)}$ $CD = \frac{h}{\tan x} \sqrt{2 - 2 \cos y}$	$\checkmark \tan x = \frac{h}{CB}$ $\checkmark CB = \frac{h}{\tan x}$ $\checkmark \text{Correct application of cos rule/}$ $\text{korrekte gebruik van cosreël}$ $\checkmark \text{substitution}$ $\checkmark \text{simplify/}$ vereenvoudig $\checkmark \text{factorising/ faktoriseer}$
	[6]

QUESTION/VRAAG 8



8.1.1	$\hat{Q}_2 = \hat{R}_2$ (tangents from same point)/ <i>raaklyne uit dsde punt</i> $2\hat{R}_2 + 70^\circ = 180^\circ$ ($\angle$ 'sof ΔPQR / $\angle$ 'e van ΔPQR) $2\hat{R}_2 = 110^\circ$ $\hat{R}_2 = 55^\circ$	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark$ answer/ <i>antwoord</i>	(4)
8.1.2	$\hat{Q}_1 + \hat{Q}_2 = 90^\circ$ (tangent $\perp$ radius)/(<i>raaklyn $\perp$ op radius</i>) $\hat{Q}_1 + 55^\circ = 90^\circ$ $\hat{Q}_1 = 35^\circ$	$\checkmark S \checkmark R$ $\checkmark$ answer / <i>antwoord</i>	(3)
8.1.3	$\hat{Q}_1 = \hat{R}_1 = 35^\circ$ ($r =$) $\hat{M}_1 = 110$ ($\angle$ 'sof ΔMQR) $\hat{M}_2 = \hat{M}_1 = 110$ (= chords subtend = $\angle$'s at centre)/ (= <i>koorde = middelpunts $\angle$'e</i>)	$\checkmark S/R$ $\checkmark \hat{M}_2 = 110$ $\checkmark R$	(3)

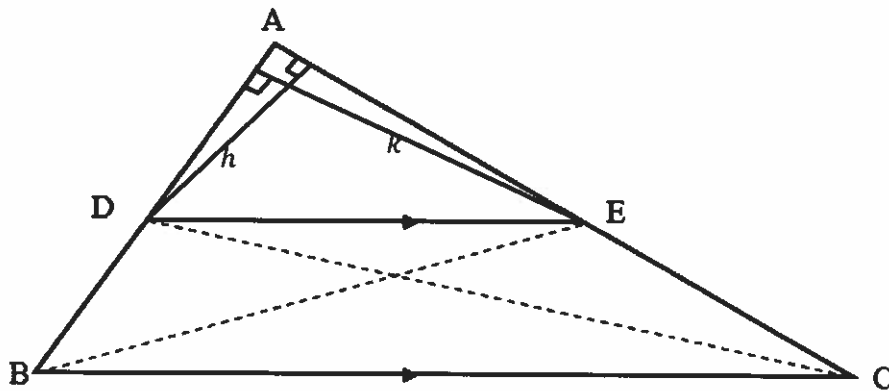
8.2



8.2.1	$\hat{X}_1 = 90^\circ$ (line from centre to midpoint of chord)/ (van midpunt van $\odot$, mdpt van koord) $\hat{U}_3 + \hat{U}_4 = 90^\circ$ ($\angle$ in semi $\odot$) $\therefore \hat{X}_1 = \hat{U}_3 + \hat{U}_4 = 90^\circ$ $\Rightarrow RU \parallel SV$ (corresponding $\angle$'s =)/ (ooreenkomstige $\angle$ 'e =)	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S$ $\checkmark R$	(5)
8.2.2	$\hat{R}_2 = y$ (tan-chord theorem)/ ($\angle$ tussen raaklyn en koord) $\hat{O}_1 = y$ (alternate $\angle$'s, $RU \parallel SV$)/ verwisselende $\angle$ 'e, $RU \parallel SV$) $\hat{O}_1 = 2\hat{T}_1$ ($\angle$ at centre = $2\angle$ at circumference) (middelpunts $\angle$ = $2 \times$ omtreks $\angle$) $\therefore \hat{T}_1 = \frac{1}{2}y$	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S \checkmark R$	(5)
			[20]

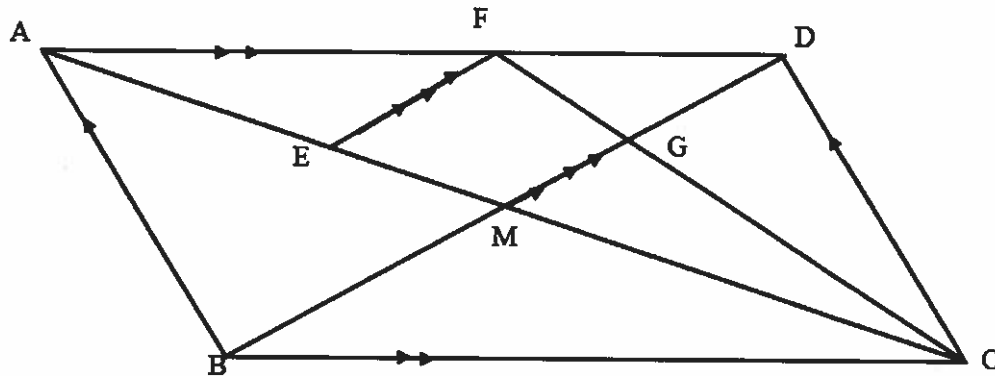
QUESTION/VRAAG 9

9.1



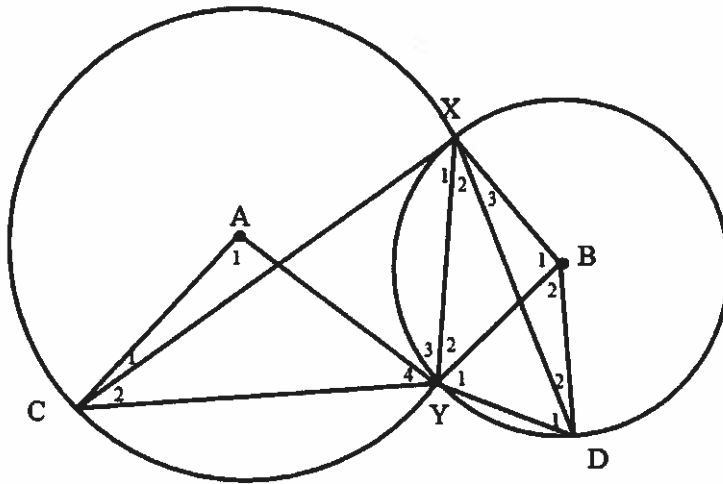
9.1	Construction: Draw altitudes h and k. <i>Konstruksie: trek hoogtelyne h en k.</i> $\frac{\text{area}\triangle ADE}{\text{area}\triangle BDE} = \frac{\frac{1}{2} \times AD \times h}{\frac{1}{2} \times BD \times h} = \frac{AD}{DB}$ $\frac{\text{area}\triangle ADE}{\text{area}\triangle DEC} = \frac{\frac{1}{2} \times AE \times k}{\frac{1}{2} \times EC \times k} = \frac{AE}{EC}$ But the $\text{area}\triangle BDE = \text{area}\triangle DEC$ (same base and same height/ <i>dsde basis en hoogte</i>) $\therefore \frac{\text{area}\triangle ADE}{\text{area}\triangle BDE} = \frac{\text{area}\triangle ADE}{\text{area}\triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓Construction/ <i>konstruksie</i> ✓S ✓S ✓S✓R ✓S (6)
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9.2



9.2.1	$\frac{AE}{EM} = \frac{AF}{FD} = \frac{4}{3}$ (line $\parallel$ to one side of $\triangle AMD$) / (lyn $\parallel$ aan een sy van $\triangle AMD$) $\frac{EM}{AM} = \frac{3}{7}$	$\checkmark S \checkmark R$ $\checkmark$ answer (3)
9.2.2	$AM : MC = 7 : 7$ (diagonals of parm) $\frac{CM}{ME} = \frac{7}{3}$	$\checkmark S \checkmark R$ $\checkmark$ answer (3)
9.2.3	$area\triangle BDC = area\triangle ABD$ (diagonals of parm) $\frac{area\triangle FDC}{area\triangle BDC} = \frac{area\triangle FDC}{area\triangle ABD}$ $= \frac{\frac{1}{2} \times FD \times height}{\frac{1}{2} \times AD \times height}$ $= \frac{FD}{AD}$ (between same parallel lines) / (tussen dieselfde $\parallel$ lyne) $= \frac{3}{7}$	$\checkmark S$ $\checkmark$ replacing / vervang $\checkmark$ area formula / oppervlak formule $\checkmark R$ (4)
		[16]

QUESTION /VRAAG 10



10.1	In $\triangle XYC$ and $\triangle DYX$: $\hat{X}_1 = \hat{D}_1$ (tan-chord theorem/ $\angle$ tussen raaklyn en koord) $\hat{C}_2 = \hat{X}_2$ (tan-chord theorem/ $\angle$ tussen raaklyn en koord) $\hat{Y}_3 + \hat{Y}_4 = \hat{Y}_1 + \hat{Y}_2$ ($\angle$'sof Δ) $\therefore \triangle XYC \parallel \triangle DYX$ ($\angle \angle \angle$) $\Rightarrow \frac{XY}{DY} = \frac{YC}{YX}$ $\therefore XY^2 = YC \cdot DY$	✓S✓R ✓S ✓S ✓S (5)
10.2	$\hat{A}_1 = 2\hat{X}_1$ ($\angle$ at centre is twice $\angle$ at circumference) (middelpunts $\angle = 2 \times$ omtreks $\angle$) Similarly/netso $\hat{B}_1 = 2\hat{D}_1$ But $\hat{X}_1 = \hat{D}_1$ (tan-chord theorem/ $\angle$ tussen raaklyn en koord) $\therefore \hat{A}_1 = \hat{B}_1$	✓S✓R ✓ ✓S (4)
10.3	$\frac{CA}{YB} = \frac{CY}{YX}$ ($\triangle CAY \parallel \triangle YBX$) $\frac{R}{r} = \frac{CY}{YX}$ $\therefore \frac{R^2}{r^2} = \frac{CY^2}{YX^2}$ $= \frac{CY \cdot CY}{CY \cdot DY}$ $= \frac{CY}{DY}$	✓S ✓Substitute/ vervang ✓Square both sides/ kwadreer albei kante

	$\therefore \frac{r^2}{R^2} = \frac{DY}{CY}$	✓Replace/vervang YX^2 ✓answer /antwoord (5)
		[14]