



GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINSIALE EKSAMEN
NOVEMBER 2023
GRAAD 11
NASIENRIGLYNE

WISKUNDE (VRAESTEL 2)

14 bladsye

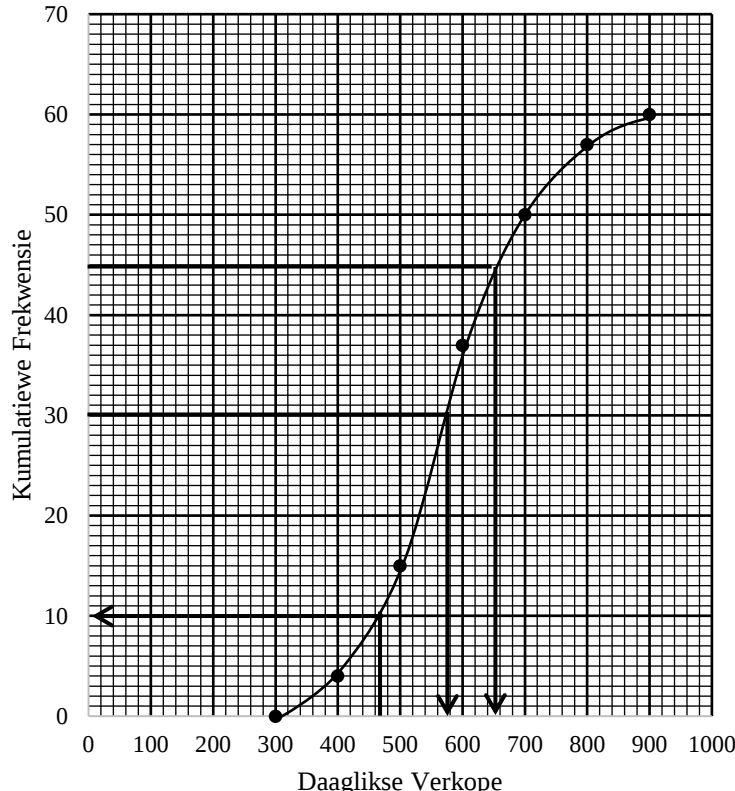
INSTRUKSIES EN INLIGTING

- **A** – AKKURAAATHEID
- **CA** – KONTINUE AKKURAAATHEID
- **S** – BEWERING
- **R** – REDE
- **S & R** – BEWERING met REDE

NOTAS:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en dit nie weer gedoen het nie, merk die doodgetrekte antwoord.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyne toegepas word.
- Dit is ONAANVAARBAAR om waardes/antwoorde aan te neem om probleme op te los.

VRAAG 1

1.1	Klas interval	Frekwensie	Kumulatiewe frekwensie	✓ Frekwensie waardes ✓ Kumulatiewe frekwensie waardes	
	$300 < x \leq 400$	4	4		
	$400 < x \leq 500$	11	15		
	$500 < x \leq 600$	22	37		
	$600 < x \leq 700$	13	50		
	$700 < x \leq 800$	7	57		
	$800 < x \leq 900$	3	60		(2)
1.2				✓ (300 ; 0) ✓ Eindpunte ✓ Vorm	(3)
1.3	$Q_2 = R580$			✓✓ Antwoord Aanvaar R560 tot R600	(2)
1.4	Op 10 dae maak hul 'n verlies. ∴ 50 dae maak hul 'n wins.			✓✓ Antwoord	(2)
1.5	Om in die top 25% van die vorige maande te wees, moet die snoepwinkel se verkope meer as R650 wees. Daarom sal R725 in die top 25% wees.			✓ R650 ✓ Gevolgtrekking	(2)
					[11]

VRAAG 2

2.1	$\frac{2k + k + 1 + k + 2 + k - 3 + 2k - 2}{5} = 15$ $\frac{7k - 2}{5} = 15$ $7k - 2 = 75$ $7k = 77$ $k = 11$	✓ $7k - 2$ ✓ Deel deur 5 ✓ Manipulasie	(3)
2.2	22 ; 12 ; 13 ; 8 ; 20 $\sigma = 5,22$	✓✓ Antwoord	(2)
2.3	Gemiddeld neem af met t eenhede maar die standaardafwyking word nie geaffekteer nie.	✓ Gemiddeld neem af met t eenhede ✓ Standaardafwyking nie geaffekteer nie	(2)
			[7]

VRAAG 3

3.1	$m_{MQ} = \frac{0 - (-2)}{-2 - (-4)}$ $m_{MQ} = -\frac{1}{3}$	✓ Substitusie in die korrekte formule ✓ Antwoord	(2)
3.2	$m_{MQ} \times -\frac{1}{3} = -1 \quad MP \perp MQ$ $m_{MP} = 3$ $y = 3x + 6$	✓ Loodregte lyne, produk van gradiënte ✓ Gradiënt van MP ✓ Antwoord	(3)
3.3	$-2 = \frac{x_R + 4}{2} \quad 0 = \frac{y_R + (-2)}{2}$ $x_R = -8 \quad y_R = 2$ $R(-8; 2)$	✓ Substitusie in die middelpunt formule ✓ x_R ✓ y_R	(3)

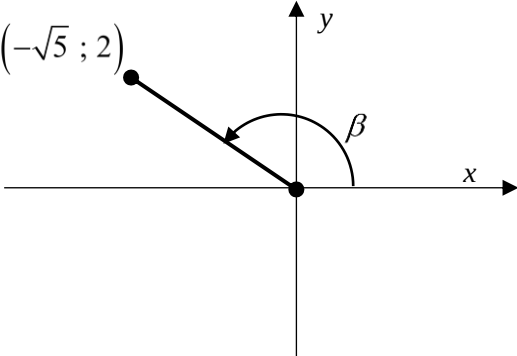
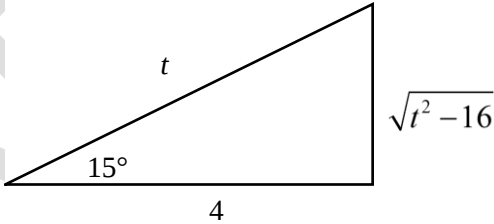
3.4	$PQ = \sqrt{(0-4)^2 + (6-(-2))^2}$ $PQ = \sqrt{80}$ $PQ = 4\sqrt{5}$	✓ Substitusie in die afstandsformule ✓ Vereenvoudigde wortelvorm	(2)
3.5	$E(-4; -6)$	✓ $x_E = -4$ ✓ $y_E = -6$	(2)
3.5	Die hoeklyne van die parallelogram is loodreg op mekaar.	✓ Hoeklyne loodreg ✓ Gegee parallelogram	(2)
			[14]

VRAAG 4

4.1	subst. $x = 0$ $py - x - 5p = 0$ $py - (0) = 5p$ $y = \frac{5p}{p}$ $y = 5$ $C(0; 5)$	✓ $x = 0$ ✓ Manipulasie ✓ $x = 5$	(3)
4.2	$py - x - 5p = 0$ $py = x + 5p$ $y = \frac{x}{p} + 5$ $m_{AC} = \frac{1}{2}$ $\therefore \frac{1}{p} = \frac{1}{2}$ $\therefore p = 2$ OF $A(-10; 0)$ $m_{AC} = \frac{1}{2}$ $p(0) - (-10) - 5p = 0$ $10 = 5p$ $p = 2$	✓ Standaardvorm ✓ Stel gradiënte gelyk ✓ $A(-10; 0)$ ✓ Vervang A	(2)

4.3	Vergelyking van AC: $2y - x - 10 = 0$ $y = \frac{1}{2}x + 5$ Vergelyking van BD: $y = -2x - 10$ $\frac{1}{2}x + 5 = -2x - 10$ $x + 10 = -4x - 20$ $5x = -30$ $x = -6$ $y = 2$ $\therefore B(-6; 2)$	✓ Vervang $p = 2$ ✓ Vergelyking van BD ✓ Vervang y in die ander vergelyking ✓ $x = -6$ ✓ $y = 2$	(5)
4.4	$\tan \alpha = \frac{1}{2}$ $\alpha = 26,57^\circ$	✓ Vervang in die korrekte formule ✓ Antwoord	(2)
4.5	$\hat{A}CD = 63,43^\circ$ binne $\angle$ e van Δ $\beta = 26,56^\circ$ binne $\angle$ e van Δ $\therefore \alpha = \beta$ $\therefore ABOD$ is 'n koordevierhoek (omgekeerde $\angle$ e in dieselfde sirkelsegment)	✓ $\hat{A}CD$ ✓ β ✓ Rede	(3)
4.6	DB is 'n middellyn (lyn onderspan 90°) $M_{CD} = \left(\frac{0+0}{2}; \frac{5+(-10)}{2} \right)$ $= M_{CD} \left(0; -\frac{5}{2} \right)$	✓ $x = 0$ ✓ $y = -\frac{5}{2}$	(2)
			[17]

VRAAG 5

5.1	5.1.1	$\sin \beta = \frac{2}{3}$ $x^2 + (2)^2 = 3^2$ $x = -\sqrt{5}$ $3\cos^2 \beta - 1$ $= 3\left(\frac{\sqrt{5}}{3}\right)^2 - 1$ $= \frac{5}{3} - 1$ $= \frac{2}{3}$ 	✓ Kwadrant II ✓ $x = -\sqrt{5}$ ✓ Vervang $\cos \beta$ ✓ Antwoord	(4)
	5.1.2	$\tan (-\beta - 180^\circ)$ $= -\tan \beta$ $= -\left(\frac{2}{-\sqrt{5}}\right)$ $= \frac{2}{\sqrt{5}}$	✓ $-\tan \beta$ ✓ Substitutie ✓ Antwoord	(3)
5.2	5.2.1	$t \cos 15^\circ = 4$ $\cos 15^\circ = \frac{4}{t}$ $\sin 15^\circ = \frac{\sqrt{t^2 - 16}}{t}$ 	✓ $\cos 15^\circ = \frac{t}{4}$ ✓ $\sqrt{t^2 - 16}$ ✓ t	(3)
	5.2.2	$\sin 75^\circ$ $= \cos 15^\circ$ $= \frac{4}{t}$	✓ Komplement ✓ Antwoord	(2)

5.2.3	$1 - \tan^2 15^\circ$ $= 1 - \left(\frac{\sqrt{t^2 - 16}}{4} \right)^2$ $= 1 - \frac{t^2 - 16}{16}$ $= \frac{16 - t^2 + 16}{16}$ $= \frac{32 - t^2}{16}$	$\checkmark \left(\frac{\sqrt{t^2 - 16}}{4} \right)^2$ $\checkmark \frac{t^2 - 16}{16}$ $\checkmark \text{ Korrekte vermenigvuldiging met KGN}$	(3)	
5.3	$\frac{\cos(90^\circ - \alpha) \sin(-\alpha - 540^\circ)}{\tan 225^\circ + \sin \alpha \cdot \sin(180^\circ + \alpha)}$ $= \frac{\sin \alpha \cdot \sin \alpha}{\tan 45^\circ + (\sin \alpha) \cdot (-\sin \alpha)}$ $= \frac{\sin^2 \alpha}{1 - \sin^2 \alpha}$ $= \frac{\sin^2 \alpha}{\cos^2 \alpha}$ $= \tan^2 \alpha$	$\checkmark \tan 45^\circ$ $\checkmark \sin \alpha \cdot \sin \alpha$ $\checkmark \sin \alpha \cdot \sin \alpha$ $\checkmark 1 - \sin^2 \alpha$ $\checkmark \frac{1}{\tan^2 \alpha}$	(5)	
5.4	5.4.1	$1 - \cos \theta = 2 \sin^2 \theta$ $1 - \cos \theta = 2 (1 - \cos^2 \theta)$ $1 - \cos \theta = 2 - 2 \cos^2 \theta$ $2 \cos^2 \theta - \cos \theta - 1 = 0$ $(2 \cos \theta + 1)(\cos \theta - 1) = 0$	$\checkmark 2 (1 - \cos^2 \theta)$ $\checkmark \text{ standaardvorm}$	(2)
5.4.2	$2 \cos \theta + 1 = 0 \quad \text{of} \quad \cos \theta - 1 = 0$ $\cos \theta = -\frac{1}{2} \quad \text{of} \quad \cos \theta = 1$ $\text{RA} = 60^\circ \quad \text{of} \quad \theta = 0^\circ + \text{K} \cdot 360^\circ ; \text{K} \in \mathbb{Z}$ $\text{QII} : \theta = 120^\circ + \text{K} \cdot 360^\circ ; \text{K} \in \mathbb{Z}$ $\text{QIII} : \theta = 240^\circ + \text{K} \cdot 360^\circ ; \text{K} \in \mathbb{Z}$	$\checkmark \cos \theta = -\frac{1}{2}$ $\checkmark \cos \theta = 1$ $\checkmark \text{ Beide oplossings vir } \cos \theta = -\frac{1}{2}$ $\checkmark \theta = 0^\circ + \text{K} \cdot 360^\circ$ $\checkmark \text{K} \in \mathbb{Z}$	(5)	
[27]				

VRAAG 6

6.1	180°	✓ Antwoord	(1)
6.2	$-\frac{1}{2} \leq y \leq \frac{1}{2}$	✓ Eindpunte ✓ Notasie	(2)
6.3.1	$45^\circ \leq x \leq 90^\circ$	✓ $x \geq 45^\circ$ ✓ $x < 90^\circ$	(2)
6.3.2	$-90^\circ < x < -45^\circ$ of $90^\circ < x < 135^\circ$	✓ $-90^\circ < x < -45^\circ$ ✓ $90^\circ < x < 135^\circ$	(2)
6.3.3	$2 \tan x + \cos 2x = 2$ $2 \tan x - 2 = -\cos 2x$ $\tan x - 1 = -\frac{1}{2} \cos 2x$ $\therefore f(x) = g(x)$ waar $x = 45^\circ$	✓ $2 \tan x - 2 = -\cos 2x$ ✓ $\tan x - 1 = -\frac{1}{2} \cos 2x$ ✓ Waarde van x	(3)
6.4	$h(x) = \frac{\sin x + \cos x}{\cos x}$ $h(x) = \frac{\sin x}{\cos x} + 1$ $h(x) = \tan x + 1$ $h(x) = f(x) + 2$ $\therefore$ Vertikale translasië op et 2 eenhede	✓ $\tan x + 1$ ✓ Op met 2 eenhede	(2)
6.5	$p(x) = 4 \left(-\frac{1}{2} \cos 2x \right)$ $p(x) = -2 \cos 2x$ $\therefore$ Maksimum waarde van 2	✓ $-2 \cos 2x$ ✓ Maks van 2	(2)
			[14]

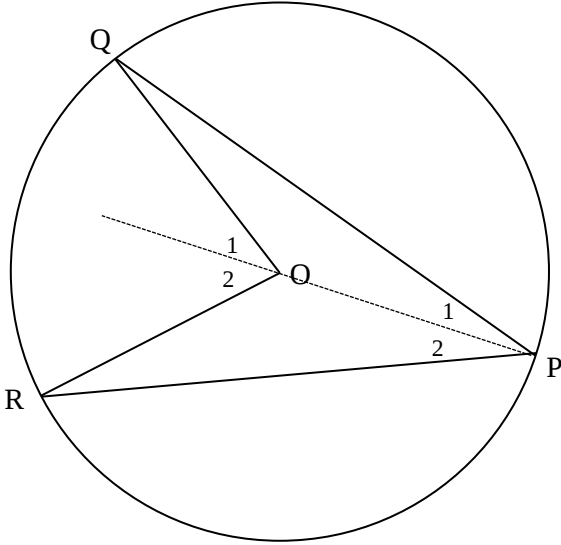
VRAAG 7

7.1	$\tan p = \frac{1}{3}$ $p = 18,435^\circ$	✓ $\tan p = \frac{1}{3}$ ✓ Waarde van p tot 3 desimale plekke	(2)
7.2	In $\triangle ABCD$ $\frac{CD}{\sin\left(\frac{18,4^\circ}{2}\right)} = \frac{121}{\sin 135^\circ}$ $CD = \frac{121 \times \sin 9,2^\circ}{\sin 135^\circ}$ $CD = 27,36 \text{ m}$	✓ Korrekte substitusie in die sinreël ✓ Manipulasie ✓ Lengte van CD	(3)
			[5]

VRAAG 8

8.1	$V = x^2 h$ $490 = x^2 h$ $h = \frac{490}{x^2}$	✓ Substitusie in die volume formule ✓ h in terme van x	(2)
8.2	$A = x^2 + 4xh$ $A = x^2 + 4x\left(\frac{490}{x^2}\right)$ $A = x^2 + \frac{1960}{x} \text{ cm}^3$	✓ Basis van x^2 ✓ $4x\left(\frac{490}{x^2}\right)$	(2)
8.3	$10 = \frac{490}{x^2}$ $x^2 = 49$ $x = 7 \quad (x > 0)$ $A = (7)^2 + \frac{1 \ 960}{7}$ $A = 329 \text{ cm}^2$	✓ Vervang $h = 10$ ✓ $x^2 = 49$ ✓ $x = 7$ ✓ Buite opp	(4)
			[8]

VRAAG 9

9.1	 Verbind PO en verleng In $\triangle POQ$ $\hat{O}_1 = \hat{Q} + \hat{P}_1$ maar $\hat{Q} = \hat{P}_1$ $\therefore \hat{O}_1 = 2\hat{P}_1$ Netso in $\triangle ROQ$ $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{P}_1 + 2\hat{P}_2$ $\hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ $\therefore \hat{ROQ} = 2\hat{P}$ buite $\angle$ van Δ $\angle$e teenoor gelyke radiusse			✓ Konstruksie ✓ SR ✓ SR ✓ S ✓ S ✓ S	(6)
9.2	9.2.1				
	(a)	$\hat{D} = 55^\circ$	$\angle$ e in dieselfde segment	✓ S ✓ R	(2)
	(b)	$\hat{O}_2 = 110^\circ$	middelpunt $\angle = 2 \times$ omtreks $\angle$	✓ S ✓ R	(2)
	(c)	$\hat{E}_2 = 110^\circ - 90^\circ$ $\hat{E}_2 = 20^\circ$	buite $\angle$ van Δ	✓ R ✓ S	(2)

	(d)	$\hat{E}_1 = 35^\circ$ buite $\angle e$ van Δ $\hat{E}_3 = 35^\circ$ $\angle e$ in 'n halwe sirkel OF $\hat{E}_3 = \hat{F}_1$ $\angle e$ teenoor gelyke radiusse $\hat{E}_3 = 35^\circ$ binne $\angle e$ van Δ	$\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ SR	(3)
9.2.2	OE = 5 HE = 4,7 $OH^2 = 5^2 - (4,7)^2$ OH = 1,71 eenhede OF OE = 5 $\sin 20^\circ = \frac{OH}{5}$ OH = 1,71 eenhede OF HE = 4,7 $\sin 20^\circ = \frac{OH}{4,7}$ OH = 1,71 eenhede	gelyke radiusse loodlyn van middelpunt van koord Pythagoras radiusse loodlyn van middelpunt na koord	$\checkmark$ SR $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ SR $\checkmark \checkmark \sin 20^\circ$ $\checkmark$ OH $\checkmark$ S $\checkmark$ R $\checkmark \tan 20^\circ$ $\checkmark$ OH	(4)
				[19]

VRAAG 10

10.1	$\hat{B}_1 = x$ $\hat{V}_1 = x$ $\hat{T}_2 = x$	verw. $\angle e$, $AC \parallel TV$ $\angle e$ teenoor gelyke sye rklyn koord stelling	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(6)
10.2	$\hat{A} = 2x$ $\hat{B}_3 = 2x$ $\therefore \hat{A} = \hat{B}_3$ $AT = BT$	teenoorst $\angle e$ van parm buite $\angle$ van kvh albei gelyk aan $2x$ sye teenoor gelyke $\angle e$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R	

	OF $\hat{B}_3 = 2x$ buite $\angle$ van kvh $\hat{T}_1 = 2x$ verw. $\angle$ e, $AC \parallel TV$ $\therefore \hat{T}_1 = \hat{V}_1 + \hat{V}_2$ albei gelyk aan $2x$ $\therefore$ BTVC is 'n gelykbenige trapezium (een paar ewewydige sye en een paar basis hoeke gelyk) $\therefore CV = BT$ sye van gelykbenige trap en $CV = AT$ teenoorst. sye parm $\therefore AT = BT$	$\checkmark$ S $\checkmark$ R $\checkmark$ SR $\checkmark$ S $\checkmark$ R $\checkmark$ Logika	(5)
10.3	In ΔATB , $5x = 180^\circ$ binne $\angle$ e van Δ $x = 36^\circ$	$\checkmark$ R $\checkmark$ S	(2)
			[13]

VRAAG 11

11.1	$\hat{D}_3 = \hat{H}_1 + \hat{E}_1$ buite $\angle$ van Δ	$\checkmark$ S	(1)
11.2	$\hat{D}_3 = \hat{H}_1 + \hat{E}_1$ buite $\angle$ van Δ $\hat{H}_1 = \hat{F}_2$ rklyn koord stelling en $\hat{E}_1 = \hat{F}_1$ rklyn koord stelling $\therefore \hat{D}_3 = \hat{F}_1 + \hat{F}_2$ $\therefore$ DEFH is 'n kvh (buite $\angle$ = teenoorst binne $\angle$)	$\checkmark$ S $\checkmark$ R $\checkmark$ SR $\checkmark$ R (met $(\hat{D}_3 = \hat{F}_1 + \hat{F}_2)$)	(4)
11.3	$\hat{H}_1 = \hat{F}_2$ rklyn koord stelling $\hat{H}_1 = \hat{F}_1$ $\angle$ e in dieselfde sirkelsegment $\therefore \hat{F}_1 = \hat{F}_2$	$\checkmark$ SR $\checkmark$ S $\checkmark$ R	(3)

