

(1) Simplify, without using a calculator: (Write your answer as a positive exponent!)

$$\begin{aligned}
 \text{(a)} \quad & \left(\frac{125x^{-3}}{343y^{-3}} \right)^{-\frac{2}{3}} \\
 &= \left(\frac{5^3 x^{-3}}{7^3 y^{-3}} \right)^{-\frac{2}{3}} \\
 &= \frac{(5^3)^{-\frac{2}{3}} (x^{-3})^{-\frac{2}{3}}}{(7^3)^{-\frac{2}{3}} (y^{-3})^{-\frac{2}{3}}} \\
 &= \frac{5^{-2} x^2}{7^{-2} y^2} \\
 &= \frac{7^2 x^2}{5^2 y^2} = \frac{49x^2}{25y^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{4 \cdot 8^{m+1} + 2^m \cdot 4^{m+1}}{32^m \cdot 4^{1-2m}} \\
 &= \frac{4 \cdot (2^3)^{m+1} + 2^m \cdot (2^2)^{m+1}}{(2^5)^m \cdot (2^2)^{1-2m}} \\
 &= \frac{2^2 \cdot 2^{3m+3} + 2^m \cdot 2^{2m+2}}{2^{5m} \cdot 2^{2-4m}} \\
 &= \frac{2^{2+3m+3} + 2^{m+2m+2}}{2^{5m+2-4m}} \\
 &= \frac{2^{3m} \cdot 2^5 + 2^{3m} \cdot 2^2}{2^m \cdot 2^2} \\
 &= \frac{2^{3m} (2^5 + 2^2)}{2^m \cdot 2^2} \\
 &= \frac{2^{3m-m} (32 + 4)}{4} \\
 &= 2^{2m} \left(\frac{32+4}{4} \right) \\
 &= 2^{2m} \left(\frac{36}{4} \right) \\
 &= 2^{2m} (9) \\
 &= 9 \cdot 2^{2m}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & \frac{25^{\frac{1}{2}} \times 32^{\frac{-1}{5}} + 27^{\frac{1}{3}} \div 8^{\frac{1}{3}}}{18^0 \times 0,5^{-2}} \\
 = & \frac{(5^2)^{\frac{1}{2}} \times (2^5)^{\frac{-1}{5}} + (3^3)^{\frac{1}{3}} \div (2^3)^{\frac{1}{3}}}{18^0 \times \left(\frac{1}{2}\right)^{-2}} \\
 = & \frac{5^1 \times 2^{-1} + 3^1 \div 2^1}{1 \times (2^{-1})^{-2}} \\
 = & \frac{5 \times \frac{1}{2} + \frac{3}{2}}{1 \times 2^2} \\
 = & \frac{\frac{5}{2} + \frac{3}{2}}{1 \times 4} \\
 = & \frac{\frac{8}{2}}{4} \\
 = & \frac{4}{4} = 1
 \end{aligned}$$

(2) Solve for x:

$$(a) \quad \sqrt[3]{8x^9} = 0,5^{-3}$$

$$\sqrt[3]{2^3 x^9} = \left(\frac{1}{2}\right)^{-3}$$

$$2^{\frac{3}{3}} x^{\frac{9}{3}} = (2^{-1})^{-3}$$

$$2^1 x^3 = 2^3$$

$$x^3 = \frac{2^3}{2^1}$$

$$x^3 = 2^2$$

$$\left(x^{\frac{3}{3}}\right)^{\frac{1}{3}} = \left(2^2\right)^{\frac{1}{3}}$$

$$x = 2^{\frac{2}{3}}$$

$$\text{or } x = \sqrt[3]{2^2} = \sqrt[3]{4}$$

$$(b) (2^{x+1} - 3)(2^{x+1} + 3) = 4^x - 6$$

$$(2^{x+1})^2 - (3)^2 = 4^x - 6$$

$$2^{2x+2} - 9 = 2^{2x} - 6$$

$$2^{2x+2} - 2^{2x} = -6 + 9$$

$$2^{2x} \cdot 2^2 - 2^{2x} = 3$$

$$2^{2x} (4 - 1) = 3$$

$$2^{2x} (3) = 3$$

$$2^{2x} = \frac{3}{3}$$

$$2^{2x} = 1$$

$$2^{2x} = 2^0$$

$$EB \Leftrightarrow EE$$

$$\therefore 2x = 0$$

$$x = 0$$

$$(3) \text{ Prove that: } \frac{m^{-2} - n^{-2}}{m^{-2}n - mn^{-2}} = \frac{m+n}{n^2 + mn + m^2}$$

$$LHS = \frac{m^{-2} - n^{-2}}{m^{-2}n - mn^{-2}}$$

$$= (m^{-2} - n^{-2}) \div (m^{-2}n - mn^{-2})$$

$$= \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \div \left(\frac{n}{m^2} - \frac{m}{n^2} \right)$$

$$= \left(\frac{n^2 - m^2}{m^2 n^2} \right) \div \left(\frac{n^3 - m^3}{m^2 n^2} \right)$$

$$= \frac{n^2 - m^2}{m^2 n^2} \times \frac{m^2 n^2}{n^3 - m^3}$$

$$= \frac{(n-m)(n+m)}{m^2 n^2} \times \frac{m^2 n^2}{(n-m)(n^2 + nm + m^2)}$$

$$= \frac{n+m}{n^2 + nm + m^2}$$

$$= \frac{m+n}{m^2 + mn + n^2} = RHS$$