Worksheet 2015-10ƵB-00009(4) – Algebraic fractions

(1) Factorise completely:

(a) $\frac{2}{(k-1)^2} - \frac{4}{k^2-1} - \frac{5}{*(1-k)^2} + \frac{3}{1+k}$

$= \frac{2}{(k-1)(k-1)} - \frac{4}{(k-1)(k+1)} - \frac{5}{(1-k)(1-k)} + \frac{3}{(1+k)}$

$= \frac{2}{(k-1)(k-1)} - \frac{4}{(k-1)(k+1)} - \frac{5}{(-1)(k-1)(-1)(k-1)} + \frac{3}{(k+1)}$

$= \frac{2}{(k-1)(k-1)} - \frac{4}{(k-1)(k+1)} - \frac{5}{(k-1)(k-1)} + \frac{3}{(k+1)}$

$= \frac{2(k+1) - 4(k-1) - 5(k+1) + 3(k-1)(k+1)}{(k-1)(k-1)(k+1)}$

$= \frac{2k+2-4k+4-5k-5+3(k^2-1)}{(k-1)(k-1)(k+1)}$

$= \frac{-7k+1+3k^2-3}{(k-1)(k-1)(k+1)}$

$= \frac{3k^2-7k-2}{(k-1)(k-1)(k+1)}$

$* \quad (1-k)^2 = (1-k)(1-k)$

$= (-1)(k-1)(-1)(k-1)$

$= (-1)(-1)(k-1)(k-1)$

$= (+1)(k-1)^2$

$\therefore \quad (1-k)^2 = (k-1)^2$

(b) $\dfrac{ab + 2a}{ab - 2a} \div \left(\dfrac{b^2 + 2b}{b + 3} \times \dfrac{b^2}{ab^2 - 3ab + 2a}\right)$

$$= \frac{\cancel{a}(b+2)}{\cancel{a}(b-2)} \div \left(\frac{b(b+2)}{(b+3)} \times \frac{b^2}{a(b^2-3b+2)}\right)$$

$$= \frac{(b+2)}{(b-2)} \div \left(\frac{b(b+2)}{(b+3)} \times \frac{b^2}{a(b-2)(b-1)}\right)$$

$$= \frac{(b+2)}{(b-2)} \div \frac{b^3(b+2)}{a(b+3)(b-2)(b-1)}$$

$$= \frac{(\cancel{b+2})}{(\cancel{b-2})} \times \frac{a(b+3)(\cancel{b-2})(b-1)}{b^3(\cancel{b+2})}$$

$$= \frac{a(b+3)(b-1)}{b^3}$$

(c) $\dfrac{\dfrac{2}{x-2} + \dfrac{1}{x-1}}{\dfrac{3}{x^2-2x} + \dfrac{1}{x}}$

$$= \left(\frac{2}{x-2} + \frac{1}{x-1}\right) \div \left(\frac{3}{x(x-2)} + \frac{1}{x}\right)$$

$$= \left(\frac{2}{(x-2)} + \frac{1}{(x-1)}\right) \div \left(\frac{3 + 1(x-2)}{x(x-2)}\right)$$

$$= \left(\frac{2(x-1) + 1(x-2)}{(x-2)(x-1)}\right) \div \left(\frac{3 + 1x - 2}{x(x-2)}\right)$$

$$= \left(\frac{2x - 2 + 1x - 2}{(x-2)(x-1)}\right) \div \left(\frac{(x+1)}{x(x-2)}\right)$$

$$= \frac{(3x-4)}{(\cancel{x-2})(x-1)} \times \frac{x(\cancel{x-2})}{(x+1)}$$

$$= \frac{x(3x-4)}{(x-1)(x+1)}$$

(d) $\dfrac{m}{m-2} + \dfrac{m^2-4}{m} \times \dfrac{m^2+2m}{m^2+4m+4} - \dfrac{5m}{m-1} \div \dfrac{10m^2}{m^2-1}$

$= \dfrac{m}{(m-2)} + \dfrac{(m-2)(m+2)}{m} \times \dfrac{m(m+2)}{(m+2)(m+2)} - \dfrac{5m}{(m-1)} \times \dfrac{m^2-1}{10m^2}$

$= \dfrac{m}{(m-2)} + \dfrac{(m-2)\cancel{(m+2)}}{\cancel{m}} \times \dfrac{\cancel{m}\cancel{(m+2)}}{\cancel{(m+2)}\cancel{(m+2)}} - \dfrac{\cancel{5m}}{\cancel{(m-1)}} \times \dfrac{\cancel{(m-1)}(m+1)}{\underset{2}{\cancel{10}}\cancel{m} \times m}$

$= \dfrac{m}{(m-2)} + \dfrac{(m-2)}{1} - \dfrac{(m+1)}{2m}$

$= \dfrac{m \times 2m + (m-2) \times 2m \times (m-2) - (m+1)(m-2)}{2m(m-2)}$

$= \dfrac{2m^2 + 2m(m-2)(m-2) - (m^2-2m+1m-2)}{2m(m-2)}$

$= \dfrac{2m^2 + 2m(m^2-2m-2m+4) - (m^2-1m-2)}{2m(m-2)}$

$= \dfrac{2m^2 + 2m(m^2-4m+4) - m^2+1m+2}{2m(m-2)}$

$= \dfrac{2m^2 + 2m^3 - \cancel{8m} + \cancel{8m} - m^2 + 1m + 2}{2m(m-2)}$

$= \dfrac{2m^3 + 1m^2 + 1m + 2}{2m(m-2)}$

(2) If $x + \frac{1}{x} = 5$, calculate without using a calculator, the value of:

[Hint: Square the given and use factorisation!]

$$\left(x + \frac{1}{x}\right)^2 = (5)^2$$

$$\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 25$$

$$x^2 + x \times \frac{1}{x} + \frac{1}{x} \times x + \frac{1}{x^2} = 25$$

$$x^2 + 1 + 1 + \frac{1}{x^2} = 25$$

$$\therefore \quad x^2 + \frac{1}{x^2} = 25 - 1 - 1 = 23$$

(a) $x^2 + \frac{1}{x^2}$

$= 23$

(b) $x^3 + \frac{1}{x^3}$ → Sum of 2 cubes!

$= \left(x + \frac{1}{x}\right)\left(x^2 - x \times \frac{1}{x} + \frac{1}{x^2}\right)$

$= \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - \frac{\cancel{x}}{1} \times \frac{1}{\cancel{x}}\right)$

$= (5)(23 - 1)$

$= (5)(22)$

$= 110$