

## MEMO 2024-12 EF-00002(1) Inverses

(1) Determine  $f^{-1}(x)$  in each of the following and write it in the form  $f^{-1}(x) = \dots\dots$

(a)  $f(x) = 3x - 4 \rightarrow y = 3x - 4$

$$f^{-1}: \quad x = 3y - 4$$

$$\therefore 3y = x + 4$$

$$\therefore y = \frac{x+4}{3}$$

$$\therefore f^{-1}(x) = \frac{x+4}{3}$$

(b)  $f(x) = 5^x \rightarrow y = 5^x$

$$f^{-1}: \quad x = 5^y$$

$$\therefore y = \log_5 x$$

$$\therefore f^{-1}(x) = \log_5 x$$

(c)  $f(x) = -2x^2 \rightarrow y = -2x^2$

$$f^{-1}: \quad x = -2y^2$$

$$\therefore y^2 = \frac{x}{-2}$$

$$\therefore y = \pm \sqrt{-\frac{x}{2}}$$

$$\therefore f^{-1}(x) = y = \pm \sqrt{-\frac{x}{2}}$$

(d)  $f(x) = \log_{0,5} x \rightarrow y = \log_{0,5} x$

$$f^{-1}: \quad x = \log_{0,5} y$$

$$\therefore y = 0,5^x$$

$$\therefore f^{-1}(x) = 0,5^x$$

(2) Determine  $g^{-1}(x)$  in each of the following and write it in the form  $g^{-1}: x \rightarrow \dots\dots$

(a)  $g: x \rightarrow \frac{x}{4} \rightarrow y = \frac{x}{4}$

$$g^{-1}: \quad x = \frac{y}{4}$$

$$\therefore y = 4x$$

$$\therefore g^{-1}: x \rightarrow 4x$$

(b)  $g: x \rightarrow \log_3 x \rightarrow y = \log_3 x$

$$g^{-1}: \quad x = \log_3 y$$

$$\therefore y = 3^x$$

$$\therefore g^{-1}: x \rightarrow 3^x$$

(c)  $g: x \rightarrow 3^{x+1} \rightarrow y = 3^{x+1}$

$$g^{-1}: \quad x = 3^{y+1}$$

$$\therefore y + 1 = \log_3 x$$

$$\therefore y = \log_3 x - 1$$

$$\therefore g^{-1}: x \rightarrow \log_3 x - 1$$

(d)  $g: x \rightarrow -0,5x \rightarrow y = -0,5x$

$$g^{-1}: \quad x = -0,5y$$

$$\therefore y = \frac{x}{-0,5}$$

$$\therefore g^{-1}: x \rightarrow -\frac{x}{0,5}$$

(3) Determine  $h$  in each of the following and write it in the form  $h(x) = \dots\dots$

(a)  $h^{-1}(x) = \log_7 x \rightarrow y = \log_7 x$

$h: x = \log_7 y$

$\therefore y = 7^x$

$\therefore h(x) = 7^x$

(b)  $h^{-1}(x) = \frac{x-2}{3} \rightarrow y = \frac{x-2}{3}$

$h: x = \frac{y-2}{3}$

$\therefore 3x = y - 2$

$\therefore y = 3x + 2$

$\therefore h(x) = 3x + 2$

(c)  $h^{-1}(x) = \frac{1}{4}x^2 \rightarrow y = \frac{1}{4}x^2$

$h: x = \frac{1}{4}y^2$

$\therefore 4x = y^2$

$\therefore y = \pm\sqrt{4x}$

$\therefore h(x) = \pm\sqrt{4x}$

(d)  $h^{-1}(x) = \log x \rightarrow y = \log x$

$h: x = \log y$

$\therefore y = 10^x$

$\therefore h(x) = 10^x$

(4) Consider the following:  $p(x) = \{(1; 7); (2; 8); (3; 9); (4; 10)\}$

(a) Is  $p$  a function? Motivate your answer.

**Yes,  $p$  is a function because none of the  $x$ -coordinates are repeated.**

(b) Write down the range of  $p^{-1}(x)$ .

$R_{p^{-1}} = \{1; 2; 3; 4\} \rightarrow R_{p^{-1}} = D_p$

(5) Explain the difference between  $f^{-1}(x)$  and  $(f(x))^{-1}$ .

**$f^{-1}(x)$  is the inverse of a function and  $(f(x))^{-1}$  is the reciprocal of the function.**

$\therefore$  If  $f(x) = 2x$ , then  $f^{-1}(x) = \frac{x}{2}$  and  $(f(x))^{-1} = \frac{1}{2x}$ .

$\rightarrow y = 2x \rightarrow$  For  $f^{-1}$ :  $x = 2y$