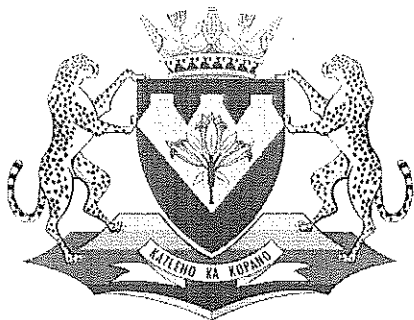




education

Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION
VOORBEREIDENDE EKSAMEN

GRADE/GRAAD 12

MATHEMATICS P1
WISKUNDE VI

SEPTEMBER 2021

MARKS/PUNTE: 150

MARKING GUIDELINES
NASIENRIGLYNE

These marking guidelines consists of 19 pages.
Hierdie nasienriglyne bestaan uit 19 bladsye.

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

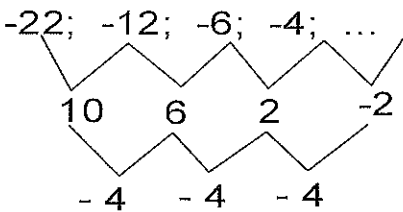
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

QUESTION/VRAAG 1

1.1.1	$x^2 - 4x - 21 = 0$ $(x-7)(x+3) = 0$ $x = 7$ or/of $x = -3$	✓ factors ✓ $x = 7$ ✓ $x = -3$	(3)
1.1.2	$x(5x-1) = 3$ $5x^2 - x - 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-3)}}{2(5)}$ $= \frac{1 \pm \sqrt{61}}{10}$ $x = 0,88$ or/of $x = -0,68$ OR/OF $5x^2 - x - 3 = 0$ $x^2 - \frac{1}{5}x + \frac{1}{100} = \frac{3}{5} + \frac{1}{100}$ $\left(x - \frac{1}{10}\right)^2 = \frac{61}{100}$ $x - \frac{1}{10} = \frac{\pm\sqrt{61}}{10}$ $x = \frac{1 \pm \sqrt{61}}{10}$ $x = 0,88$ or/of $x = -0,638$	✓ standard form ✓ substitution into correct formula ✓ $x = 0,88$ ✓ $x = -0,68$ OR/OF ✓ standard form ✓ $x^2 - \frac{1}{5}x + \frac{1}{100} = \frac{3}{5} + \frac{1}{100}$ ✓ $x = 0,88$ ✓ $x = -0,68$	(4)
1.1.3	$2x^2 - 9x + 4 \geq 0$ $(2x-1)(x-4) \geq 0$ $x \leq \frac{1}{2}$ or/of $x \geq 4$	✓ factors ✓✓ answers (combo marks)	(3)
1.1.4	$3^{x+1} - 3^{x-1} - 24 = 0$ $3^x(3 - 3^{-1}) = 24$ $3^x\left(\frac{8}{3}\right) = 24$ $3^x = 3^2$ $\therefore x = 2$	✓ $3^x(3 - 3^{-1}) = 24$ ✓ $3^x\left(\frac{8}{3}\right) = 24$ ✓ $3^x = 3^2$ ✓ answer	(4)

1.2	$y + 2x = 2$ and/en $y^2 - 3yx = -2x^2$ $y = 2 - 2x$ $y^2 - 3yx = -2x^2$ $(2 - 2x)^2 - 3x(2 - 2x) = -2x^2$ $4 - 8x + 4x^2 - 6x + 6x^2 = -2x^2$ $12x^2 - 14x + 4 = 0$ $6x^2 - 7x + 2 = 0$ $(3x - 2)(2x - 1) = 0$ $\therefore x = \frac{2}{3}$ or/of $x = \frac{1}{2}$ $\therefore y = 2 - 2\left(\frac{2}{3}\right)$ or/of $y = 2 - 2\left(\frac{1}{2}\right)$ $y = \frac{2}{3}$ $y = 1$	$\checkmark y = 2 - 2x$ $\checkmark$ substitution $\checkmark$ standard form $\checkmark$ both x values $\checkmark$ both y values
1.3	$\left(\sqrt[4]{\sqrt{20} - \sqrt{D_x(4x)}}\right)\left(\sqrt[4]{\sqrt{20} + \sqrt{D_x(4x)}}\right)$ $= \left(\sqrt[4]{\sqrt{20} - \sqrt{4}}\right)\left(\sqrt[4]{\sqrt{20} + \sqrt{4}}\right)$ $= \left(\sqrt[4]{\sqrt{20} - 2}\right)\left(\sqrt[4]{\sqrt{20} + 2}\right)$ $= \sqrt[4]{20 - 4}$ $= \sqrt[4]{16}$ $= 2$	$\checkmark \left(\sqrt[4]{\sqrt{20} - \sqrt{4}}\right)\left(\sqrt[4]{\sqrt{20} + \sqrt{4}}\right)$ $\checkmark \left(\sqrt[4]{\sqrt{20} - 2}\right)\left(\sqrt[4]{\sqrt{20} + 2}\right)$ $\checkmark \sqrt[4]{20 - 4}$ $\checkmark$ answer
		(5)
		(4)
		[23]

QUESTION/VRAAG 2

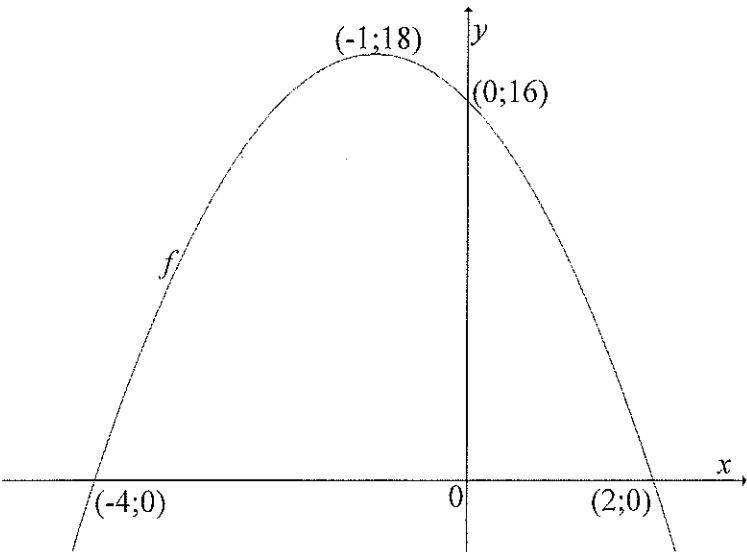
2.1.1	 The next two terms are/die volgende twee terme is -6 and/en -12	✓ -6 ✓ -10 (2)
2.1.2	$2a = -4 \quad 3(-2) + b = 10 \quad -2 + 16 + c = -22$ $\therefore a = -2 \quad \therefore b = 16 \quad \therefore c = -36$ $\therefore T_n = -2n^2 + 16n - 36$ Answer only: Full marks/ Slegs antwoord: Volpunte	✓ $a = -2$ ✓ $b = 16$ ✓ $c = -36$ ✓ $T_n = -2n^2 + 16n - 36$ (4)
2.1.3	$\therefore T_n = -2n^2 + 16n - 36$ $n = -\frac{b}{2a}$ $= -\frac{16}{2(-2)}$ $n = 4$ Answer only: Full marks/ Slegs antwoord: Volpunte OR/OF $T'_n = -4n + 16$ $0 = -4n + 16$ $\therefore n = 4$	✓ substitution ✓ answer OR/OF ✓ $0 = -4n + 16$ ✓ answer (2)
2.2.1	$d = -6$	✓ answer (1)
2.2.2	$T_n = 27 - 6(n + 1)$ $-117 = 27 - 6(n + 1)$ $6n = 138$ $\therefore n = 23$	✓ $-117 = 27 - 6(n + 1)$ ✓ answer (2)
2.3.1	$5 + 9 + 13 + \dots$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_n = \frac{n}{2}[2(5) + (n - 1)4]$ $= \frac{n}{2}[10 + 4n - 4]$ $S_n = 2n^2 + 3n$	✓ Substitution into the correct formula ✓ answer (2)

2.3.2	$S_n = 2n^2 + 3n$ $\therefore S_{n-6} = 2(n-6)^2 + 3(n-6)$ $= 2n^2 - 24n + 72 + 3n - 18$ $= 2n^2 - 21n + 54$ OR/OF $S_{n-6} = S_n - 906$ $= 2n^2 + 3n - 906$	$\checkmark S_{n-6} = 2(n-6)^2 + 3(n-6)$ $\checkmark S_{n-6} = 2n^2 - 21n + 54$ (2) OR/OF $\checkmark S_{n-6} = S_n - 906$ $\checkmark S_{n-6} = 2n^2 + 3n - 906$ (2)
2.3.3	$906 = S_n - S_{n-6}$ $906 = 2n^2 + 3n - (2n^2 - 21n + 54)$ $906 = 24n - 54$ $960 = 24n$ $\therefore n = 40$	$\checkmark$ setting up the equation $\checkmark$ substitution $\checkmark$ simplification ($960 = 24n$) $\checkmark$ answer (4)
		[19]

QUESTION/VRAAG 3

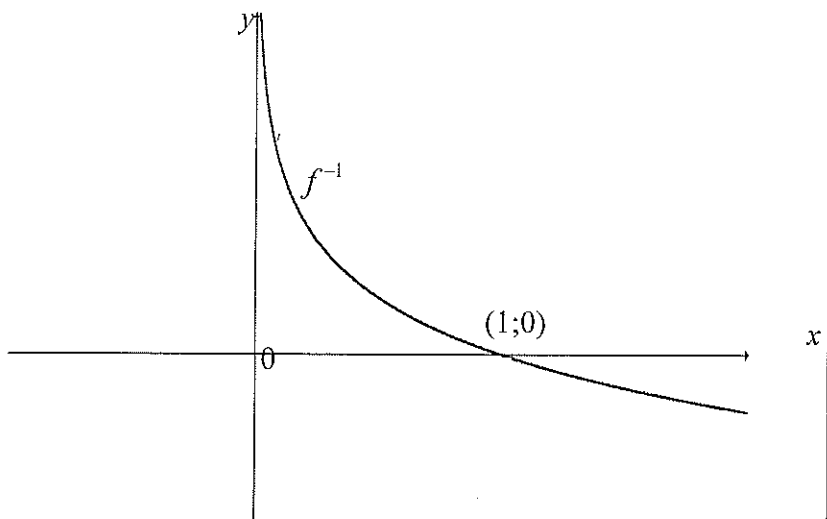
3.1	$T_n > \frac{3}{16384}$ $ar^{n-1} > \frac{3}{16384}$ $3\left(\frac{1}{2}\right)^{n-1} > \frac{3}{16384}$ $\left(\frac{1}{2}\right)^{n-1} > \left(\frac{1}{2}\right)^{14} \quad \text{or/of} \quad 2^{-n+1} > 2^{-14}$ $\therefore n-1 < 14 \qquad \therefore -n+1 > -14$ $n < 15 \qquad n < 15$ $\therefore n = 14 \qquad \therefore n = 14$ OR/OF $1; 2; 4; \dots$ $ar^{n-1} < 16384$ $1 \cdot 2^{n-1} < 16384$ $2^{n-1} < 2^{14}$ $\therefore n-1 < 14$ $n < 15$ $\therefore n = 14$	✓ substituting into the correct formula ✓ method/same base ✓ calculating n ✓ answer (4)
3.2	$\sum_{n=1}^{30} 3\left(\frac{1}{2}\right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \dots$ $\sum_{k=1}^{\infty} 27p^k = 27p + 27p^2 + 27p^3 + \dots$ $\frac{a}{1-r} = \frac{a(1-r^n)}{1-r}$ $\frac{27p}{1-p} = \frac{3\left(1-\left(\frac{1}{2}\right)^{30}\right)}{1-\frac{1}{2}}$ $\frac{27p}{1-p} = 6$ $27p = 6 - 6p$ $\therefore p = \frac{2}{11}$	✓ Both expansions ✓ Substitution into S_{∞} formula ✓ Substitution into S_n formula ✓ simplification: $\frac{27p}{1-p} = 6$ ✓ answer (5)
		[9]

QUESTION/VRAAG 4

4.1	Y-intercept/-snypunt: $(0; 16)$	✓ y intercept: $(0; 16)$ (1)
4.2	X-intercept/-snypunt: $0 = -2x^2 - 4x + 16$ $0 = x^2 + 2x - 8$ $0 = (x + 4)(x - 2)$ $\therefore x = -4 \text{ or/of } x = 2$	✓ $y = 0$ ✓ both x values (2)
4.3	$x = -\frac{b}{2a}$ $x = -\frac{(-4)}{2(-2)} = -1$ $f(-1) = -2(-1)^2 - 4(-1) + 16$ $f(-1) = 18$ Turning point/draaipunt: $(-1; 18)$ OR/OF $f'(x) = 2x + 2$ $0 = 2x + 2$ $\therefore x = -1$ $f(-1) = -2(-1)^2 - 4(-1) + 16$ $f(-1) = 18$ Turning point/draaipunt: $(-1; 18)$	✓ method ✓ $x = -1$ ✓ Turning point : $(-1; 18)$ (3) OR/OF ✓ method ✓ $x = -1$ ✓ Turning point : $(-1; 18)$ (3)
4.4		✓ shape ✓ x and y intercepts ✓ turning point (3)

4.5	Range/ <i>waardeversameling</i> : $y \in (-\infty; 18]$	✓✓ answer (2)
	OR/OF	OR/OF
	$y \leq 18$	✓✓ answer (2)
		[11]

QUESTION/VRAAG 5

5.1	$f(x) = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $\therefore y = \log_{\frac{1}{2}} x$	$\checkmark x = \left(\frac{1}{2}\right)^y$ $\checkmark$ answer (2)
5.2	$x \in R, x > 0$ or/of $x \in (0; \infty)$	$\checkmark$ answer (1)
5.3		$\checkmark$ shape and asymptote $x = 0$ $\checkmark$ x intercept (2)
5.4	$h(x) = x + 3$ $1 = x + 3$ $\therefore x = -2$ $B(-2; 1)$	$\checkmark$ Substitution: $1 = x + 3$ $\checkmark$ answer: $B(-2; 1)$ (2)
5.5	$g(x) = \frac{a}{x+2} + 1$ $0 = \frac{a}{0+2} + 1$ $-1 = \frac{a}{2}$ $\therefore a = -2$ $g(x) = \frac{-2}{x+2} + 1$	$\checkmark$ substitute $p = 2$ and $q = 1$ $\checkmark$ substitute (0;0) $\checkmark a = -2$ $\checkmark$ answer: $g(x) = \frac{-2}{x+2} + 1$ (4)
5.6	$k(x) = \frac{-2}{x-1} + 2$	$\checkmark +2$ $\checkmark -1$ (2)
5.7.1	$-2 < x \leq 0$	$\checkmark$ critical values $\checkmark$ notation (2)

5.7.2	$x \in R; x \neq -2$	✓ ✓ answer (2)
		[17]

QUESTION/VRAAG 6

6.1	One x value is associated with two y values/<i>Een x-waarde word met twee y-waardes geassosieer.</i> OR/OF Vertical line test cuts the graph twice/<i>Vertikale lyn toets sny die grafiek twee keer.</i>	✓✓ answer OR/OF ✓✓ answer (2)
6.2	$x \leq 0$ or/of $x \geq 0$	✓ $x \leq 0$ ✓ $x \geq 0$ (2)
6.3	$y = \sqrt{\frac{x}{2}}$ $x = \sqrt{\frac{y}{2}}$ $x^2 = \frac{y}{2}$ $y = 2x^2$ $\therefore \frac{dy}{dx} = 4x$	✓ $x = \sqrt{\frac{y}{2}}$ ✓ $y = 2x^2$ ✓ $\frac{dy}{dx} = 4x$ (3)
		[7]

QUESTION/VRAAG 7

7.1	$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$ $1 + i_{eff} = \left(1 + \frac{0,12}{2}\right)^2$ $\therefore i_{eff} = 12,36\%$	✓ substitution into the correct formula ✓ answer (2)
7.2	$A = P(1 - i)^n$ $\frac{P}{2} = P(1 - 0,07)^n$ $\frac{1}{2} = (0,93)^n$ $n = \log_{0,93} \frac{1}{2}$ $\therefore n = 9,55 \text{ years/jare}$	✓ substitution into the correct formula ✓ correct use of logs ✓ answer (3)
7.3.1	$A = P(1 + i)^n$ $A = R250\,000 \left(1 + \frac{0,095}{4}\right)^{48}$ $A = R771\,343,67$	✓ $i = \frac{0,095}{4}$ and $n = 48$ ✓ substitution into the correct formula ✓ answer (3)
7.3.2	Home loan/Huislening = R2 920 000 – R771 343,67 = R2 148 656,33	✓ answer (1)
7.3.3	$i = \frac{0,103}{12} \text{ and/en } n = 240$ $P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $R2\,148\,656,33 = \frac{x \left[1 - \left(1 + \frac{0,103}{12}\right)^{-240}\right]}{\frac{0,103}{12}}$ $\therefore x = R21\,163,87$ OR/OF $F = \frac{x[(1 + i)^n - 1]}{i}$ $R2\,148\,656,33 \left(1 + \frac{0,103}{12}\right)^{240} = \frac{x \left[\left(1 + \frac{0,103}{12}\right)^{240} - 1\right]}{\frac{0,103}{12}}$ $\therefore x = R21\,163,87$	✓ $i = \frac{0,103}{12}$ and $n = 240$ ✓ substitution into the correct formula ✓ answer (3)
		OR/OF ✓ $i = \frac{0,103}{12}$ and $n = 240$ ✓ substitution into the correct formula ✓ answer (3)

7.3.4	$\begin{aligned}\text{Interest/Rente} &= \text{R}21\,163,87 \times 240 \\ &\quad - \text{R}2\,148\,656,33 \\ &= \text{R}2\,930\,672,47\end{aligned}$	✓ method ✓ answer (2)
		[14]

QUESTION/VRAAG 8

8.1	$\begin{aligned}f(x) &= -\frac{3}{x} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-\frac{3}{x+h} - \left(-\frac{3}{x}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-3x + 3(x+h)}{x(x+h)} \div h \\ &= \lim_{h \rightarrow 0} \frac{3h}{x^2 + xh} \times \frac{1}{h} \\ &= \frac{3}{x^2}\end{aligned}$	✓ substitution: $\frac{-\frac{3}{x+h} - \left(-\frac{3}{x}\right)}{h}$ ✓ expression with LCD: $\lim_{h \rightarrow 0} \frac{-3x + 3(x+h)}{x(x+h)} \div h$ ✓ simplify: $\lim_{h \rightarrow 0} \frac{3h}{x^2 + xh} \times \frac{1}{h}$ ✓ answer (4)
8.2.1	$\begin{aligned}D_x \left[(3x^3 - 2)^2 \right] \\ &= D_x [9x^6 - 12x^3 + 4] \\ &= 54x^5 - 36x^2\end{aligned}$	✓ expansion ✓ $54x^5$ ✓ $-36x^2$ (3)
8.2.2	$\begin{aligned}y &= 2x^3 - \frac{4}{x} + 4\sqrt[3]{x} \\ y &= 2x^3 - 4x^{-1} + 4x^{\frac{1}{3}} \\ \frac{dy}{dx} &= 6x^2 + 4x^{-2} + \frac{4}{3}x^{-\frac{2}{3}}\end{aligned}$	✓ $4x^{\frac{1}{3}}$ ✓ $6x^2$ ✓ $+4x^{-2}$ ✓ $+\frac{4}{3}x^{-\frac{2}{3}}$ (4)

8.3	$y = -2x + \frac{4}{3}$ $\therefore m = -2$ Gradient of tangent/<i>Gradiënt van raaklyn</i>: $f'(x) = \frac{1}{2}$ $y = 2(1-x)^2 = 2x^2 - 4x + 2$ $4x - 4 = \frac{1}{2}$ $\therefore x = \frac{9}{8}$ $\therefore y = \frac{1}{32}$ Coordinates/<i>Koördinate</i>: $\left(\frac{9}{8}, \frac{1}{32}\right)$	✓ gradient of tangent: $f'(x) = \frac{1}{2}$ ✓ $4x - 4 = \frac{1}{2}$ ✓ x value ✓ coordinates: $\left(\frac{9}{8}, \frac{1}{32}\right)$ (4)
		[15]

QUESTION/VRAAG 9

9.1	$y = a(x-1)^2(x-3)$ $6 = a(0-1)^2(0-3)$ $6 = -3a$ $\therefore a = -2$ $y = -2(x-1)^2(x-3)$ $y = -2(x^2 - 2x + 1)(x-3)$ $y = -2(x^3 - 5x^2 + 7x - 3)$ $y = -2x^3 + 10x^2 - 14x + 6$ $\therefore a = -2; b = 10; c = -14$ and/en $d = 6$	$\checkmark$ subst. $x_1 = 1$ and/en $x_2 = 3$ $\checkmark$ subst. $A(0;6)$ $\checkmark -3a = 6$ $\checkmark y = -2(x-1)^2(x-3)$ $\checkmark y = -2(x^3 - 5x^2 + 7x - 3)$
		(5)
9.2	$y = -2x^3 + 10x^2 - 14x + 6$ $f'(x) = -6x^2 + 20x - 14$ $f''(x) = -12x + 20$ $0 = -12x + 20$ $\therefore x = \frac{20}{12} = \frac{5}{3}$	$\checkmark f'(x) = -6x^2 + 20x - 14$ $\checkmark f''(x) = 0$ $\checkmark$ answer
		(3)
9.3.1	$f'(x) = -6x^2 + 20x - 14$ $0 = -6x^2 + 20x - 14$ $0 = 3x^2 - 10x + 7$ $0 = (x-1)(3x-7)$ $x = 1$ or $x = \frac{7}{3}$ $1 < x < \frac{7}{3}$ Accept/Aanvaar: $1 \leq x \leq \frac{7}{3}$	$\checkmark f'(x) = 0$ $\checkmark$ factors $\checkmark$ both x values $\checkmark\checkmark$ answer (combo mark)
		(5)
9.3.2	Turning point/Draaipunt $C\left(\frac{7}{3}; \frac{64}{27}\right)$ $\therefore 0 < k < \frac{64}{27}$	$\checkmark C\left(\frac{7}{3}; \frac{64}{27}\right)$ $\checkmark\checkmark$ answer (combo mark)
		(3)
		[16]

QUESTION/VRAAG 10

10.1	$\text{Length/Lengte } 2BC = 100 - 3x$ $\text{Length/Lengte } BC = \frac{100 - 3x}{2}$ $= 50 - \frac{3}{2}x$ $\text{Area/Oppervlakte} = AB \cdot BC$ $A(x) = x \left(50 - \frac{3}{2}x \right)$ $= 50x - \frac{3}{2}x^2$	$\checkmark \quad 2BC = 100 - 3x$ $\checkmark \quad \text{Substitution in the area formula.}$ (2)
10.2	$A'(x) = 50 - 3x$ $0 = 50 - 3x$ $\therefore x = \frac{50}{3}m$ $AB = \frac{50}{3}m$	$\checkmark \quad A'(x) = 50 - 3x$ $\checkmark \quad A'(x) = 0$ $\checkmark \quad x = \frac{50}{3}$ (3)
		[5]

QUESTION/VRAAG 11

11.1.1	$P(\text{female and green eyes})/\text{vrou met groen oë}$ $= \frac{147}{540} = \frac{49}{180} = 0,27$	✓ answer (1)
11.1.2	For independent events/Vir onafhanklike gebeure: $P(\text{female and green eyes/vroulik en groen oë}) =$ $P(\text{female/vroulik}) \times P(\text{green eyes/groen oë})$ $P(\text{female and green eyes/vroulik en groen oë}) = 0,27$ $P(\text{female/vrou/lik}) \times P(\text{green eyes/groen oë}) =$ $\frac{240}{540} \times \frac{330}{540} = \frac{22}{81} = 0,27$ $\therefore$ The events are independent, and the learner is correct/Die gebeure is onafhanklik, en die leerder is reg. OR/OF For independent events/Vir onafhanklike gebeure: $P(\text{male and green eyes/manlik en groen oë}) =$ $P(\text{male/manlik}) \times P(\text{green eyes/groen oë})$ $P(\text{male and green eyes/manlik en groen oë}) =$ $= \frac{183}{540} = \frac{61}{180} = 0,34$ $P(\text{male/manlik}) \times P(\text{green eyes/groen oë}) =$ $= \frac{300}{540} \times \frac{330}{540} = \frac{55}{162} = 0,34$ $\therefore$ The events are independent, and the learner is correct/Die gebeure is onafhanklik, en die leerder is reg.	✓ rule for independent events ✓ $P(\text{female and green eyes}) = 0,27$ ✓ $\frac{240}{540} \times \frac{330}{540} = \frac{22}{81} = 0,27$ ✓ conclusion (4) OR/OF ✓ rule for independent events ✓ $P(\text{male and green eyes}) = 0,34$ ✓ $= \frac{300}{540} \times \frac{330}{540} = \frac{55}{162} = 0,34$ ✓ conclusion (4)
11.2	$P(E \text{ and } / \text{ en } F) = P(E) \times P(F)$ $\frac{1}{3} = x \times y$ $xy = \frac{1}{3}$ $P(E \text{ or } / \text{ of } F) = P(E) + P(F) - P(E \text{ and } / \text{ en } F)$ $\frac{9}{10} = x + y - \frac{1}{3}$ $\frac{9}{10} = \frac{1}{3y} + y - \frac{1}{3}$ $27y = 10 + 30y^2 - 10y$ $37y = 30y^2 + 10$	✓ $xy = \frac{1}{3}$ ✓ $\frac{9}{10} = x + y - \frac{1}{3}$ ✓ $\frac{9}{10} = \frac{1}{3y} + y - \frac{1}{3}$ (3)
11.3.1	$13! = 6227020800$	✓✓ answer (2)
11.3.2	$7! \times 5! \times 1! = 604\,800$	✓ $7!$ ✓ $5!$

		✓ 1! ✓ 604 800 (4)
		[14]

TOTAL/TOTAAL: 150