Worksheet 2015-8EN-00003(1)

(1) Are the following statements true or false?

(a) All right- angled triangles are isosceles triangles as well.

False

(b) The sum of the interior angles of an equilateral triangle is 180°.

True

(c) The theorem of Pythagoras can only be applied in a right-angled triangle.

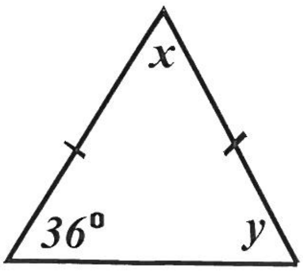
True

(d) The angles opposite the equal sides of an isosceles triangle are equal in size.

True

(2) Determine the value(s) of x, y and/or k in each of the following sketches. Also give complete reasons.

(a)



$y = 36^\circ$ [$\angle$s opp. equal sides]

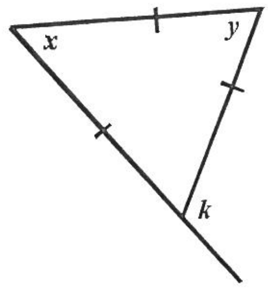
$x + y + 36^\circ = 180^\circ$ [int $\angle$s of $\triangle$]

$\therefore x + 36^\circ + 36^\circ = 180^\circ$

$\therefore x = 180^\circ - 36^\circ - 36^\circ$

$\therefore x = 108^\circ$

(b)

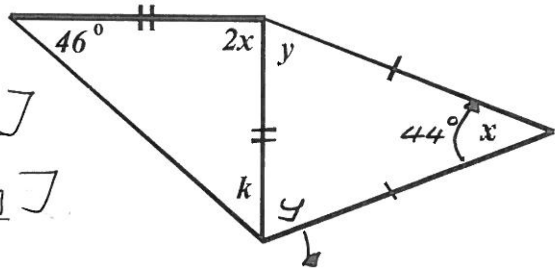


$x = y = 60°$ [$\angle^s$ of isosceles Δ]

$x + y = k$ [ext. $\angle$ of Δ]

$\therefore k = 60° + 60°$

$\therefore k = 120°$



(c)

$k = 46°$ [$\angle^s$ opp. equal sides]

$2x + 46° + k = 180°$ [int. $\angle^s$ of Δ]

$\therefore 2x = 180° - 46° - 46° = 88°$

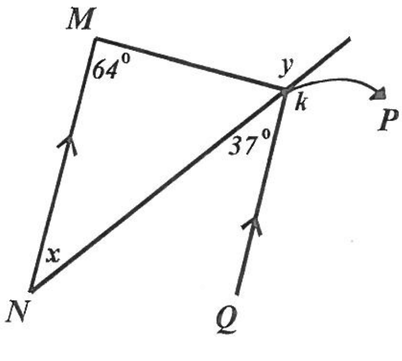
$\therefore x = \frac{88°}{2} = 44°$

$\therefore y + y + 44° = 180°$ [int. $\angle^s$ of Δ]

$2y = 180° - 44° = 136°$

$\therefore y = \frac{136°}{2} = 68°$

(d)



$x = 37°$ [alt. $\angle^s$; $MN \parallel PQ$]

$y = 64° + 37°$ [ext $\angle$ of Δ]

$\therefore y = 101°$

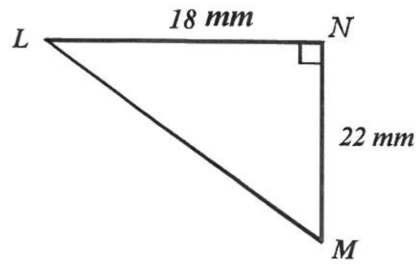
$k + 37° = 180°$ [$\angle^s$ on str. line]

$\therefore k = 180° - 37°$

$\therefore k = 143°$

(3) Calculate the length of the unknown side in each of the following:
If necessary, leave your answer in surd form.
Also calculate the area of each triangle.

(a)



$LM^2 = LN^2 + NM^2$ [Pythagoras]

$= (18)^2 + (22)^2$

$= 324 + 484$

$LM^2 = 808$

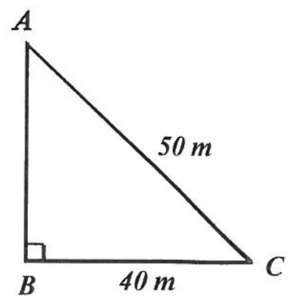
$\therefore LM = \sqrt{808}$ mm

Area $\triangle LMN = \frac{1}{2} b \times h$

$= \frac{1}{2}(18mm) \times (22 mm)$

$= 198 mm^2$

(b)



$AC^2 = AB^2 + BC^2$ [Pythagoras]

$(50)^2 = AB^2 + (40)^2$

$2500 = AB^2 + 1600$

$2500 - 1600 = AB^2$

$\therefore AB^2 = 900$

$\therefore AB = \sqrt{900}$

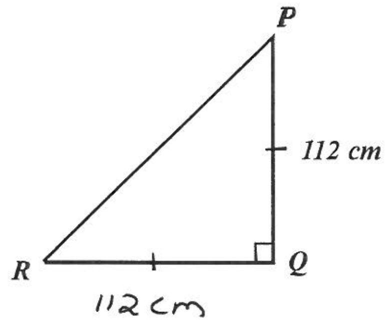
$\therefore AB = 30$ m

Area $\triangle ABC = \frac{1}{2} b \times h$

$= \frac{1}{2}(40m) \times (30m)$

$= 600 m^2$

(c)



$PR^2 = PQ^2 + RQ^2$ [Pythagoras]

$= (112)^2 + (112)^2$

$= 12\ 544 + 12\ 544$

$PR^2 = 25\ 088$

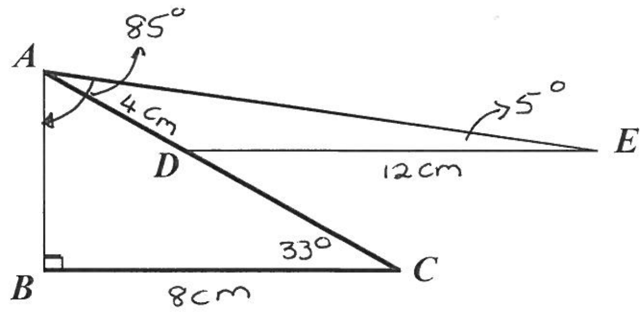
$\therefore PR = \sqrt{25\ 088}$ cm

Area $\Delta PQR = \frac{1}{2} b \times h$

$= \frac{1}{2} (112 \text{ cm}) \times (112 \text{ cm})$

$= 6\ 272 \text{ cm}^2$

(4) In the sketch: $\hat{E} = 5°$, $\hat{C} = 33°$, $B\hat{A}E = 85°$.
AC = 10 cm, BC = 8 cm ,
AD = 4 cm and DE = 12 cm.



(a) Prove that DE // BC. Show all calculations.
(b) Calculate the lengths of AB.
(c) Calculate the areas of Δ ABC and Δ ADE, if possible.

(a) $B\hat{A}C = 180° - \hat{C} - \hat{B}$ [Int. ∠s of Δ]
$= 180° - 33° - 90°$
∴ $B\hat{A}C = 57°$
∴ $D\hat{A}C = 85° - 57°$ [$B\hat{A}C = 85°$ given]
∴ $D\hat{A}C = 28°$
but $E\hat{D}C = D\hat{A}C + \hat{E}$ [ext. ∠ of Δ]
∴ $E\hat{D}C = 28° + 5°$
∴ $E\hat{D}C = 33°$
∴ $E\hat{D}C = \hat{C} = 33°$
but these are alternate ∠s
∴ DE // BC

(b) $AC^2 = AB^2 + BC^2$
$(10)^2 = AB^2 + (8)^2$
$100 = AB^2 + 64$
∴ $AB^2 = 100 - 64 = 36$
∴ $AB = \sqrt{36} = 6$ cm

(c) Area Δ ABC $= \frac{1}{2} b \times h$
$= \frac{1}{2}(8cm) \times (6cm)$
$= 24\ cm^2$

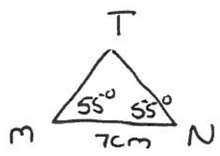
(5) (a) Construct triangles MNT with MN $= 7$ cm, $\widehat{M} = 55° = \widehat{N}$.

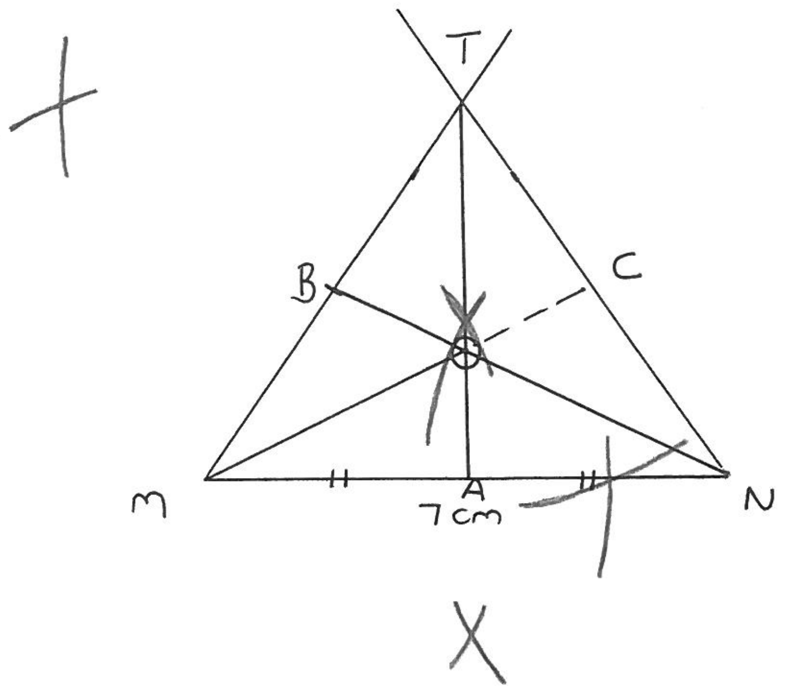
(b) Bisect MN and label the midpoint of MN as A. Join A with T.

(c) Bisect MT and label the midpoint of MT as B. Join B with N.

(d) Label the intersection of AT and BN as O.

(e) Join M and O and extend MO to intersect TN in C. Measure TC and CN.

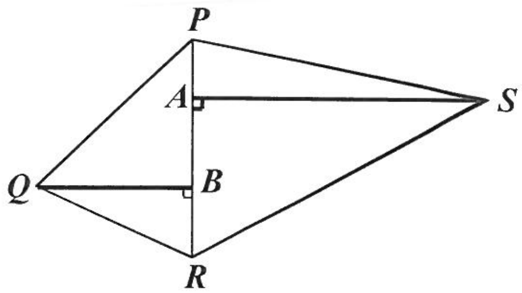




TC = 3cm

CN = 3cm

(6) Calculate the area of PQRS,
if AS $= 34$ mm, BQ $= 16$ mm,
AB $=$ BR $= 7$ mm and AP $= 6$ mm



PR = 6mm + 7mm + 7mm = 20 mm

$\therefore$ Area $\triangle$ PQR $= \frac{1}{2} b \times h$

$= \frac{1}{2} \times (RP) \times (BQ) = \frac{1}{2} \times (20mm) \times (16mm)$

$= 160\ mm^2$

Area $\triangle$ PSR $= \frac{1}{2} b \times h$

$= \frac{1}{2} (PR) \times (AS) = \frac{1}{2} \times (20mm) \times (34\ mm)$

$= 340\ mm^2$

$\therefore$ Area PQRS = area $\triangle$PQR + area $\triangle$ PSR

$= 160\ mm^2 + 340\ mm^2$

$= 500\ mm^2$