



GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINSIALE EKSAMEN

NOVEMBER 2021

GRAAD 11

NASIENRIGLYNE

WISKUNDE (VRAESTEL 2)

17 bladsye

NASIENRIGLYNE	WISKUNDE (Vraestel 2)	GRAAD 11
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INSTRUKSIES EN INLIGTING:

A – Akkuraatheid

C.A. – Kontinue Akkuraatheid (volg op)

S – Bewering

R – Rede

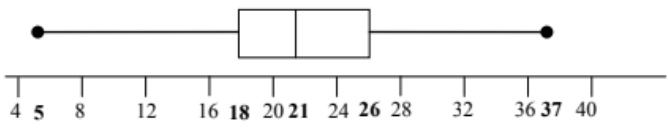
S en R – Bewering en Rede

NOTA:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en dit nie weer gedoen het nie, merk die doodgetrekte antwoord.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyne toegepas word.
- Dit is onaanvaarbaar om waardes/antwoorde aan te neem om probleme op te los.

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VRAAG 1

1.1	Gemiddeld $= \frac{\sum_{i=1}^n x_i}{n} = \frac{408}{19} = 21,47$	✓ antwoord	(1)
1.2	Standaardafwyking = 7,81	✓✓ antwoord	(2)
1.3	Die limiete vir die een standaardafwyking is ($\bar{x} - 1\sigma$; $\bar{x} + 1\sigma$) = (21,47 – 7,81; 21,47 + 7,81) = (13,66 ; 29,28) ∴ 13 mense lê binne 1 standaardafwyking van die gemiddeld af.	✓ interval ✓ 13 mense	(2)
1.4	5 12 13 15 18 18 18 19 20 21 21 22 23 23 26 29 33 35 37 IKVW = 26 – 18 = 8	✓ $Q_1 = 18$ ✓ $Q_3 = 26$ ✓ IKVW = 8	(3)
1.5		✓✓ houer / mond ✓ punt / snor	(3)
1.6	Daar is 'n merkbare verskil tussen die laagste waarde (5) en die volgende laagste waarde (12) terwyl die verskille tussen al die ander data waardes hoogstens binne 3 waardes is ∴ 5 is 'n uitskieter	✓ rede ✓ 5 is 'n uitskieter	(2)
			[13]

VRAAG 2

2.1	<table><tr><th>Lengte (cm)</th><th>Aantal leerders (f)</th><th>Kumulatiewe frekwensie</th></tr><tr><td>$145 \leq x < 150$</td><td>6</td><td>6</td></tr><tr><td>$150 \leq x < 155$</td><td>23</td><td>29</td></tr><tr><td>$155 \leq x < 160$</td><td>60</td><td>89</td></tr><tr><td>$160 \leq x < 165$</td><td>74</td><td>163</td></tr><tr><td>$165 \leq x < 170$</td><td>52</td><td>215</td></tr><tr><td>$170 \leq x < 175$</td><td>32</td><td>247</td></tr><tr><td>$175 \leq x < 180$</td><td>3</td><td>250</td></tr><tr><td>Totaal</td><td>250</td><td></td></tr></table>	Lengte (cm)	Aantal leerders (f)	Kumulatiewe frekwensie	$145 \leq x < 150$	6	6	$150 \leq x < 155$	23	29	$155 \leq x < 160$	60	89	$160 \leq x < 165$	74	163	$165 \leq x < 170$	52	215	$170 \leq x < 175$	32	247	$175 \leq x < 180$	3	250	Totaal	250		✓ 89 ✓ 215 ✓ 247	(3)
Lengte (cm)	Aantal leerders (f)	Kumulatiewe frekwensie																												
$145 \leq x < 150$	6	6																												
$150 \leq x < 155$	23	29																												
$155 \leq x < 160$	60	89																												
$160 \leq x < 165$	74	163																												
$165 \leq x < 170$	52	215																												
$170 \leq x < 175$	32	247																												
$175 \leq x < 180$	3	250																												
Totaal	250																													
2.2	Ogief wat die verspreiding van punte toon	✓ Vorm ✓ grond op x- as ✓ plot die punte	(3)																											
2.3	$Q_2 \approx 162$ $Q_1 \approx 158$ $Q_3 \approx 167$ $\therefore \text{IKVW} \approx 162$	✓ ✓ ✓ ✓	(4)																											
2.4	$165 \leq x < 170$	✓ antwoord	(1)																											
2.5	$\frac{35}{250} = 14\%$	✓ antwoord	(1)																											
2.6	$\sigma = 6,43$	✓✓ antwoord	(2)																											
			[14]																											

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VRAAG 3

3.1	$E\left(\frac{3+12}{2}; \frac{0+3}{2}\right)$ $=\left(7\frac{1}{2}; 1\frac{1}{2}\right)$	✓ substitusie in middelpuntformule ✓ Antwoord	(2)
3.2	$M_{BC} = \frac{3-0}{12-3}$ $= \frac{1}{3}$	✓ substitusie in gradiëntformule ✓ Antwoord	(2)
3.3	$\tan \theta = m_{BC} = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$	✓ $\tan \theta = m_{BC}$ ✓ antwoord	(2)
3.4	$M_{AD} = M_{BC} = \frac{1}{3}$ $AD \parallel BC$, gelyke gradiënte $M_{AB} = \frac{6-0}{1-3} = -3$ $\therefore M_{AD} \times M_{AB} = \frac{1}{3} \times -3 = -1$ $\therefore AD \perp AB$	✓ $M_{AD} = \frac{1}{3}$ ✓ $M_{AB} = -3$ ✓ $M_{AD} \times M_{AB} = -1$	(3)
3.5	Inklynasie van nuwe lyn $= 45^\circ + 18,43^\circ = 63,43^\circ$ $\therefore \tan 63,43^\circ = 2 = m_{\text{lyn}}$ $\therefore y - 6 = 2(x - 1)$ $y = 2x + 4$	✓ $18,43^\circ$ ✓ $63,43^\circ$ ✓ $m = 2$ ✓ vergelyking	(4)
			[13]

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VRAAG 4

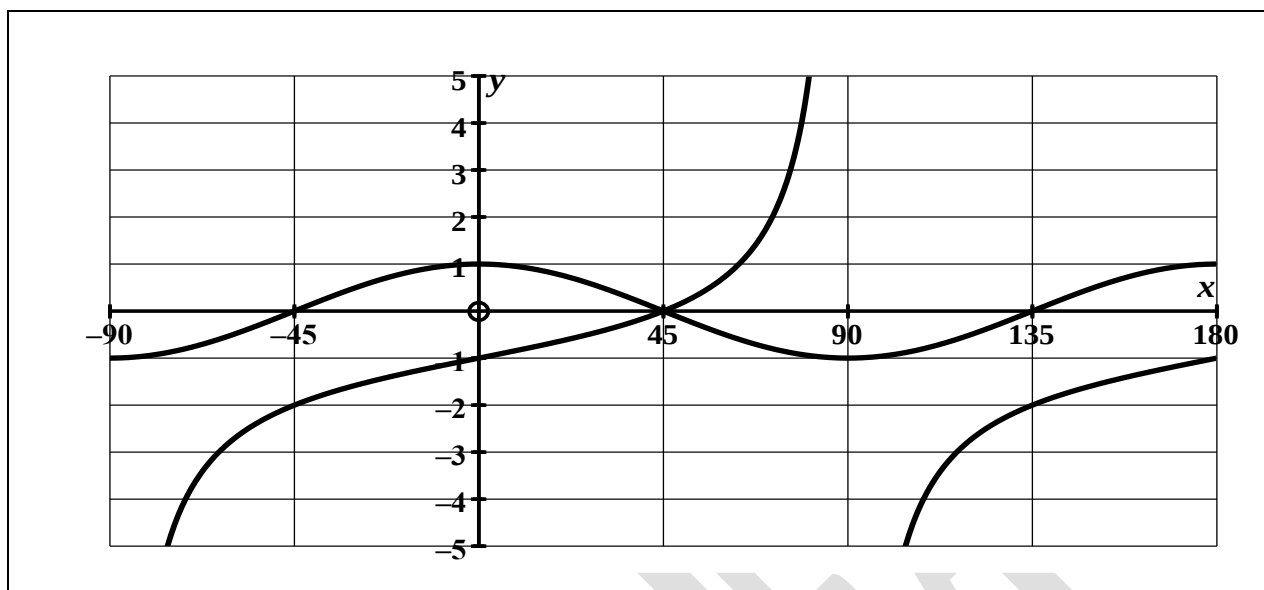
4.1	$M_{QP} = M_{OS} = 6$ $QP \parallel OS$, gelyke gradiënte $y - 17 = 6(x + 3)$ $y = 6x + 35$	$\checkmark M_{QP} = 6$ $\checkmark$ vergelyking	(2)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) - 5$ $\therefore Q(-5;5)$	$\checkmark$ opstel van vergelyking $\checkmark x = -5$ $\checkmark$ koördinate van Q	(3)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ eenhede	$\checkmark$ substitusie in afstandsformule $\checkmark 5\sqrt{2}$	(2)
4.4	$M_{OS} = 6$ $\therefore$ inklinasie van OS is $-\tan^{-1}(4) = 80,54^\circ$ $M_{OQ} = -1$ $\therefore$ inklinasie van QO is $180^\circ - \tan^{-1}(1) = 135^\circ$ $\alpha = 135^\circ - 80,54^\circ \dots$ $= 54,46^\circ$	$\checkmark 80,54^\circ$ $\checkmark 135^\circ$ $\checkmark 54,46^\circ$	(3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS \cdot OQ \cdot \cos \alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50} \cdot \cos 54,46^\circ)$ $QS = 9,90$ eenhede	$\checkmark$ korrekte gebruik van cosreël $\checkmark$ substitusie in formule $\checkmark 9,90$	(3)
			[13]

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VRAAG 5

5.1	5.1.1	$\sin 28^\circ = \cos 62^\circ$ $= k$	✓✓ Antwoord	(2)
	5.1.2	$\cos 242^\circ = -\cos 62^\circ$ $= -k$	✓✓ Antwoord	(2)
5.2		$= -\tan x \cdot \cos x = \frac{\sin x}{\cos x} \times \frac{\cos x}{\sin x}$ $-\sin x$ $= 1$	✓ $\tan x \cdot \cos x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ 1	(4)
5.3		$\frac{\cos 50^\circ - \cos \theta}{\sin 40^\circ + \cos \theta} = \frac{-(\cos \theta + \cos 50^\circ)}{\cos \theta + \sin 40^\circ}$ $\cos 50^\circ = \sin 40^\circ$ $= -1$	✓✓ $\frac{\cos 50^\circ - \cos \theta}{\sin 40^\circ + \cos \theta}$ ✓✓ $-(\cos \theta + \cos 50^\circ)$ ✓ antwoord	(5)
5.4		$= \sqrt{2^{2\sin 20^\circ} \cdot 2^{3 \cdot -\tan 45^\circ}}$ $= \sqrt{2^{2 \cdot \frac{1}{2}} \cdot 2^{-3}}$ $= \sqrt{\frac{1}{4}}$ $= \frac{1}{2}$	✓ $\sqrt{2^{2\sin 20^\circ} \cdot 2^{3 \cdot -\tan 45^\circ}}$ ✓ $\sqrt{2^{2 \cdot \frac{1}{2}} \cdot 2^{-3}}$ ✓ $\frac{1}{2}$	(3)
5.5	5.5.1	$\sin^2 x = \frac{3}{4}$ $\sin x = \pm \frac{\sqrt{3}}{2}$	✓ antwoord	(1)
	5.5.2	Verwysingshoek = 60° $60^\circ; 120^\circ; 240^\circ$	✓✓✓ antwoorde	(3)
				[20]

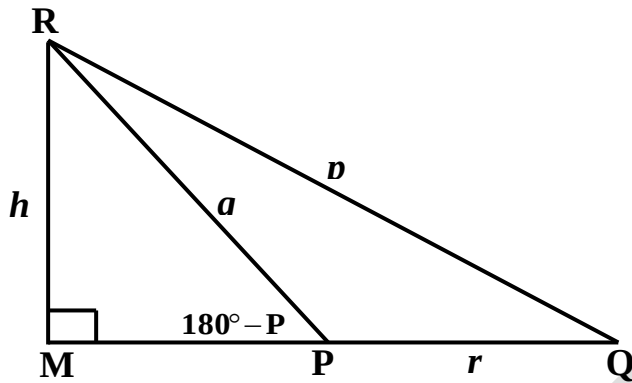
VRAAG 6



6.1	$f(x) = \tan x - 1$ $g(x) = \cos 2x$		✓ asympote ✓ afsnitte met asse ✓ vorm ✓ periode ✓ vorm met oop punt/kol by $x = -90^\circ$	(5)
6.2	6.2.1	$\cos 2x + 1 = \tan x$ $\therefore \cos 2x = \tan x - 1$ $\therefore x = 45^\circ$	✓ $x = 45^\circ$	(1)
	6.2.2	$2 \cos^2 x = 1$ $\therefore 2 \cos^2 x - 1 = 0$ $\therefore \cos 2x = 0$ $\therefore x = \pm 45^\circ$ or $x = 135^\circ$	✓ $\cos 2x = 0$ ✓ oplossings	(2)
	6.2.3	$-90^\circ < x < -45^\circ$ of $90^\circ < x < 135^\circ$	✓ $-90^\circ < x < -45^\circ$ ✓ $90^\circ < x < 135^\circ$	(2)
6.3	$h(x) = \cos 2(x - 30^\circ)$		✓ $h(x) = \cos 2(x - 30^\circ)$	(1)
				[11]

VRAAG 7

7.1



Konstruksie: Trek $RM \perp PQ$ of QP verleng

$$RM = p \sin Q$$

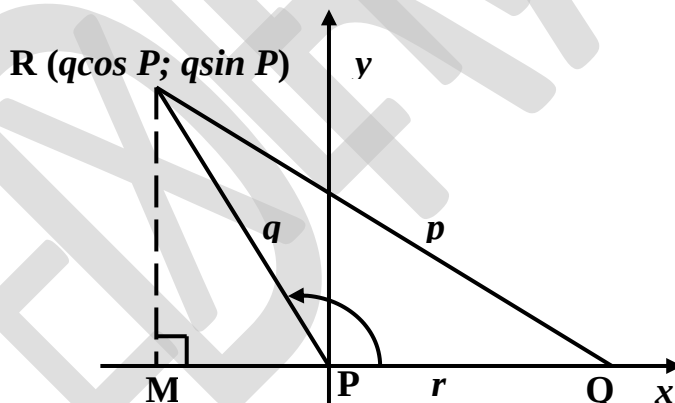
$$\begin{aligned} CM &= q \sin (180^\circ - P) \\ &= q \sin P \end{aligned}$$

$$\therefore p \sin Q = q \sin P$$

Deel deur $\sin P \cdot \sin Q$ gee dan

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

OF



P is in standaardposisie soos in die diagram getoon.

Konstruksie: Trek $RM \perp PQ$ of QP verleng

$$RM = q \sin P = p \sin Q$$

Deel deur $\sin P \cdot \sin Q$ dan kry ons

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

✓ konstruksie

$$✓ RM = p \sin Q$$

$$✓ RM = q \sin (180^\circ - P)$$

$$✓ \therefore p \sin Q = q \sin P$$

✓ Deel deur $\sin P \cdot \sin Q$

OF

$$✓ R(q \cos P; q \sin P)$$

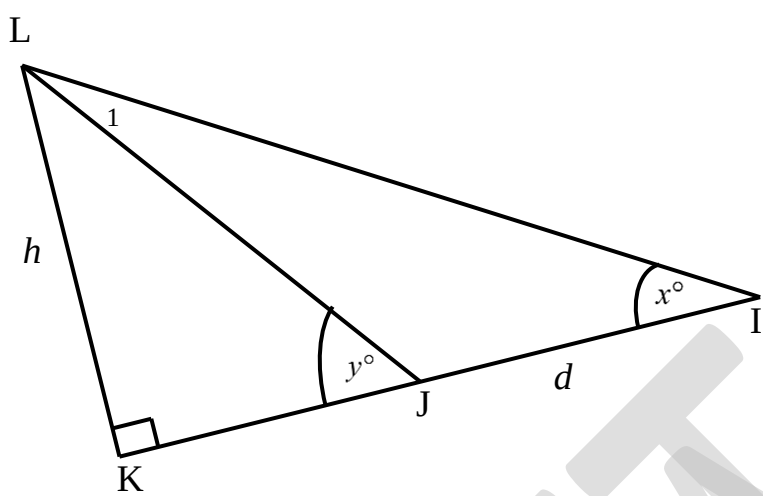
✓ konstruksie

$$✓ RM = q \sin P$$

$$✓ RM = p \sin Q$$

✓ Deel deur $\sin P \cdot \sin Q$

(5)

7.2			
7.2	7.2.1	In ΔLKJ: $\frac{h}{LJ} = \sin y$ $\therefore h = LJ \sin y \quad \text{--- (1)}$ In ΔDAB: $\frac{LJ}{\sin x} = \frac{d}{\sin \hat{L}_1}$ maar $\hat{L}_1 = y - x$ $\therefore LJ = \frac{d \sin x}{\sin(y - x)} \quad \text{--- (2)}$ sub (2) in (1): $h = \frac{d \sin x \sin y}{\sin(y - x)}$	✓ h substitusie in formule ✓ sinreël ✓ $\hat{L}_1 = y - x$ ✓ LJ ✓ substitusie (5)
	7.2.2	$h = \frac{70 \cdot \sin 12^\circ \cdot \sin 30^\circ}{\sin 18^\circ}$ $= 23,55 m$	✓ substitusie ✓ antwoord (2)
[12]			

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VRAAG 8

8.1	Volume van silinder: $V_1 = \pi r^2 h$ $= \pi (12)^2 \cdot (19)$ $= 2736 \pi \text{ m}^3$ Volume van hemisfeer (halwe sfeer): $V_2 = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$ $\text{Totale Volume} = V_1 + V_2$ $= \frac{1}{2} \left[\left(\frac{4}{3} \pi r^3 \right) (12)^3 \right]$ $= 1152 \pi \text{ m}^3$ $\text{Totale Volume} = V_1 + V_2$ $= 2736 \pi + 1152 \pi$ $= 3888 \pi \text{ m}^3$	✓ Substitusie ✓ antwoord ✓ $\frac{1}{2}$ ✓ Substitusie ✓ antwoord ✓ Totale Volume in π (5 uit 6 indien numeriese waarde van pi gebruik is)	(6)
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8.2	Buiteoppervlak van silinder: $A_1 = 2\pi r^2 + 2\pi rh$ $= 2\pi(12)^2 + 2\pi(12)(19)$ $= 2337,34 \text{ m}^2$ Buiteoppervlak van hemisfeer (halwe sfeer): $A_2 = \frac{1}{2}(4\pi r^2)$ $= \frac{1}{2}[(4\pi r^2)(12)^2]$ $= 452.39 \text{ m}^2$ <i>Totale Buiteoppervlak</i> $= A_1 + A_2$ $= 2337,34 + 452.39$ $= 685.73 \text{ m}^2 \approx 686 \text{ m}^2$	✓ Substitusie ✓ antwoord ✓ Substitusie ✓ antwoord ✓ Totale buiteoppervlak ✓ antwoord, korrek afgerond	(6) [12]
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VRAAG 9

9.1	9.1.1	Die teenoorgestelde hoeke van 'n sikliese vierhoek is supplementêr .	✓ antwoord	(1)
	9.1.2	Die hoek tussen die raaklyn en koord is gelyk aan die hoek in die teengestelde segment .	✓ antwoord	(1)
9.2	Konstrueer AF(middellyn) en FD (koord) $\hat{D}_1 = 90^\circ \quad (\angle \text{ in 'n halwe sirkel })$ $\hat{A}_1 = 90^\circ \quad (rad \perp raaklyn)$ $\hat{A}_2 = \hat{D}_2 \quad (\angle^e \text{ in dieselfde sirkelseg. })$ $\hat{A}_1 + \hat{A}_2 = \hat{D}_1 + \hat{D}_2$ $\therefore \hat{BAS} = \hat{D}$			
			✓ Konstruksie	
			✓ Bewering ✓ Rede	
			✓ Bewering en Rede	
			✓ Bewering en Rede	
			✓ Gevolgtrekking	
				(6)

9.3				
9.3.1	$\hat{PQE} = \hat{H}_3 = 70^\circ$ (raaklyn – koordstelling) $\therefore \hat{Q}_1 = 70^\circ$	✓ raaklyn – koordstelling ✓ $\hat{Q}_1 = 70^\circ$	(2)	
9.3.2	$\hat{E}_1 = 70^\circ$ (verw. $\angle e, EF \parallel QH$) $\hat{G} = 180^\circ - \hat{E}_1$ (teenoorst. $\angle e$ kvh EFGH) $= 180^\circ - 70^\circ$ $\hat{G} = 110^\circ$	✓ $\hat{E}_1 = 70^\circ$ ✓ verw. $\angle e =$ ✓ $\hat{G} = 180^\circ - \hat{E}_1$ ✓ R	(4)	
9.3.3	$\hat{Q}_2 = 180^\circ - (\hat{Q}_1 + \hat{Q}_3)$ ($\angle e$ op rtlyn PQR) $= 180^\circ - (70^\circ + 30^\circ)$ $= 80^\circ$ $\hat{EFH} = 180^\circ - \hat{Q}_2$ (teenoorst. $\angle e$ kvh EFHQ) $\hat{F}_1 = 180^\circ - 80^\circ$ $= 100^\circ$	✓ $180^\circ - (\hat{Q}_1 + \hat{Q}_3)$ ✓ $\hat{Q}_2 = 80^\circ$ ✓ teenoorst $\angle e$ van kvh ✓ $\hat{F}_1 = 100^\circ$	(4)	
			[18]	

11.2.2	$\hat{W}_1 = \hat{Q}_4$ (raaklyn koordstelling) $\hat{Q}_1 = \hat{Q}_3$ (= koorde onderspan = $\angle$) $\Rightarrow \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$ netso $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ (buite $\angle$ van Δ) $= \hat{R}_3$ (ooreenk. $\angle e, QT \parallel RS$) $\therefore \hat{R}_3 = \hat{Q}_4 + \hat{Q}_3$ PRKQ is 'n koordevierkant. (buite $\angle$ = teenoorst. binne $\angle$) Alternatiewelik: $\hat{Q}_1 + \hat{Q}_3 = \hat{Q}_3 + \hat{S}$ OF $\hat{W}_1 = \hat{Q}_4$ (raaklyn koordstelling) $\hat{Q}_1 = \hat{Q}_3$ (= koorde onderspan = $\angle$) $\therefore \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$	✓ bewering ✓ rede ✓ bewering & rede ✓ bewering & rede ✓ bewering & rede	(5)
			[12]
TOTAAL:			150