

Worksheet 2015-8EM-00006(2)

(1) Determine the value(s) of x and/or y in each of the following sketches. Also give complete reasons.

(a)

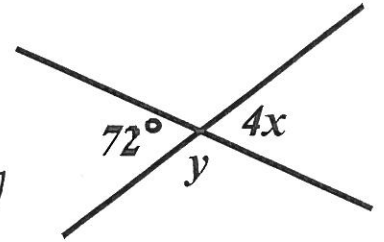
$$4x = 72^\circ \quad [\text{Vert. opp. } \angle^s]$$

$$\therefore x = \frac{72^\circ}{4} = 18^\circ$$

$$y + 72^\circ = 180^\circ \quad [\angle^s \text{ on str. line}]$$

$$\therefore y = 180^\circ - 72^\circ$$

$$\therefore y = 108^\circ$$



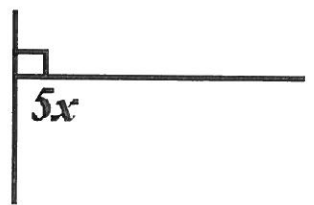
(b)

$$5x + 90^\circ = 180^\circ \quad [\angle^s \text{ on str. line}]$$

$$\therefore 5x = 180^\circ - 90^\circ$$

$$\therefore 5x = 90^\circ$$

$$\therefore x = \frac{90^\circ}{5} = 18^\circ$$



(c)

$$5y + 14^\circ + 12^\circ + 2y = 180^\circ \quad [\angle^s \text{ on str. line}]$$

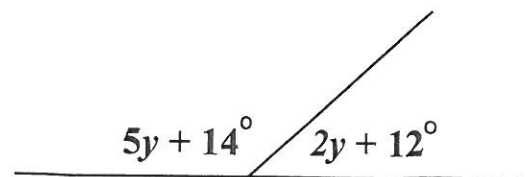
$$\therefore 7y + 26^\circ = 180^\circ$$

$$\therefore 7y = 180^\circ - 26^\circ$$

$$7y = 154^\circ$$

$$y = \frac{154^\circ}{7}$$

$$y = 22^\circ$$



(d)

$$105^\circ + 2x + 45^\circ + x + 90^\circ = 360^\circ$$

[Revolution]

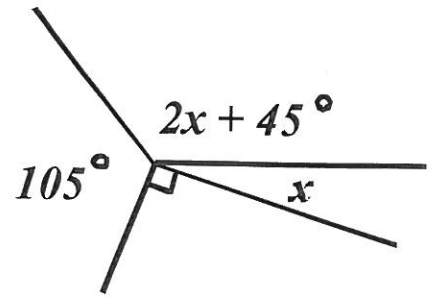
$$\therefore 3x + 240^\circ = 360^\circ$$

$$\therefore 3x = 360^\circ - 240^\circ$$

$$\therefore 3x = 120^\circ$$

$$\therefore x = \frac{120^\circ}{3}$$

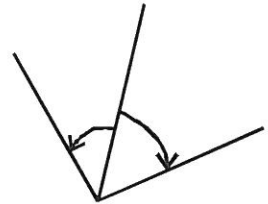
$$x = 40^\circ$$



(2) Which of the following pairs of indicated angles are adjacent? If not adjacent, motivate why not.

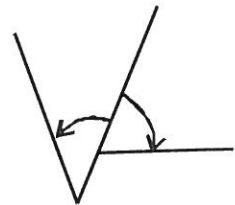
(a)

Adjacent



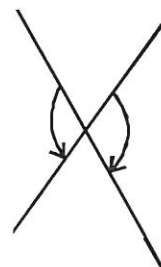
(b)

Not adjacent, because there are no common vertex



(c)

Not adjacent, because the angles do not have a common arm.



(3) Determine the value(s) of m , n and/or k in each of the following sketches. Also give complete reasons.

(a)

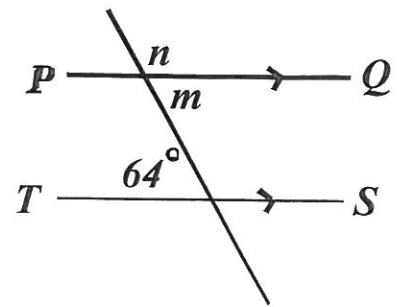
$$m = 64^\circ \text{ [alt. } \angle^s; PQ \parallel TS]$$

$$n + m = 180^\circ \text{ [} \angle^s \text{ on str. line]}$$

$$\therefore n + 64^\circ = 180^\circ$$

$$\therefore n = 180^\circ - 64^\circ$$

$$\therefore n = 116^\circ$$



(b)

$$k + 15^\circ = 180^\circ \text{ [co-int } \angle^s; XY \parallel CD]$$

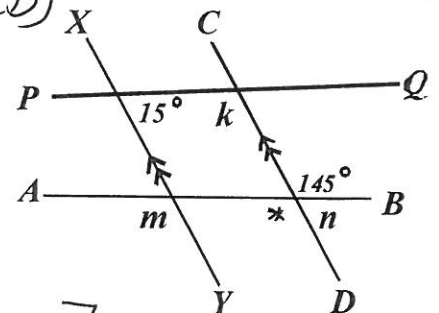
$$\therefore k = 180^\circ - 15^\circ$$

$$\therefore k = 165^\circ$$

$$n + 145^\circ = 180^\circ \text{ [} \angle^s \text{ on str. line]}$$

$$\therefore n = 180^\circ - 145^\circ$$

$$\therefore n = 35^\circ$$



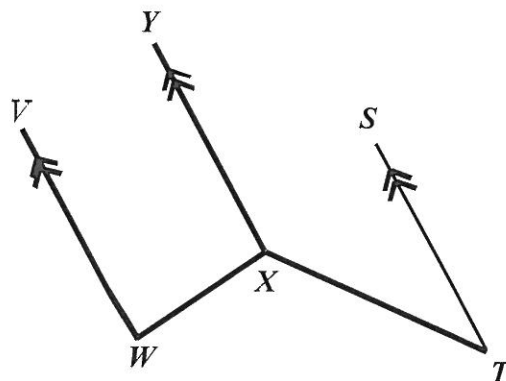
$$* = 145^\circ \text{ [Vert. opp. } \angle^s]$$

$$m = * = 145^\circ \text{ [corresp. } \angle^s; CD \parallel XY]$$

Remember: PQ not $\parallel AB$ as given!

(4) (a) Label the following angles (according to size):

- (i) $\widehat{WXT}$
- (ii) $\widehat{XTS}$
- (iii) $\widehat{TXY}$



(b) Calculate the value of x if

$$\widehat{WXT} = 154^\circ \text{ with } \widehat{TXY} = 3x + 52^\circ$$

$$\text{and } \widehat{WXY} = 4x - 14^\circ$$

(c) Calculate, with reasons: (i) $\widehat{W}$
(ii) $\widehat{T}$

- (a)(i) Obtuse angle
- (ii) Acute angle
- (iii) Obtuse angle

$$(b) \widehat{WXT} + \widehat{TXY} + \widehat{WXY} = 360^\circ \quad [\text{Revolution}]$$

$$\therefore 154^\circ + 3x + 52^\circ + 4x - 14^\circ = 360^\circ$$

$$\therefore 7x + 192^\circ = 360^\circ$$

$$\therefore 7x = 360^\circ - 192^\circ = 168^\circ$$

$$\therefore x = \frac{168^\circ}{7} = 24^\circ$$

$$(c)(i) \widehat{WXY} = 4x - 14^\circ \quad [\text{given}]$$

$$\therefore \widehat{WXY} = 4(24^\circ) - 14^\circ \quad [x = 24^\circ \rightarrow \text{see (b)}]$$

$$\therefore \widehat{WXY} = 82^\circ$$

$$\text{but } \widehat{WXY} + \widehat{W} = 180^\circ \quad [\text{co-int } \angle^s; VW \parallel XY]$$

$$\therefore 82^\circ + \widehat{W} = 180^\circ$$

$$\therefore \widehat{W} = 180^\circ - 82^\circ = 98^\circ$$

$$(ii) \widehat{YXT} = 3x + 52^\circ \quad [\text{given}]$$

$$\therefore \widehat{YXT} = 3(24^\circ) + 52^\circ \quad [x = 24^\circ \rightarrow \text{see (b)}]$$

$$\therefore \widehat{YXT} = 72^\circ + 52^\circ$$

$$\therefore \widehat{YXT} = 124^\circ$$

$$\text{but } \widehat{YXT} + \widehat{T} = 180^\circ \quad [\text{co-int } \angle^s; YX \parallel ST]$$

$$\therefore 124^\circ + \widehat{T} = 180^\circ$$

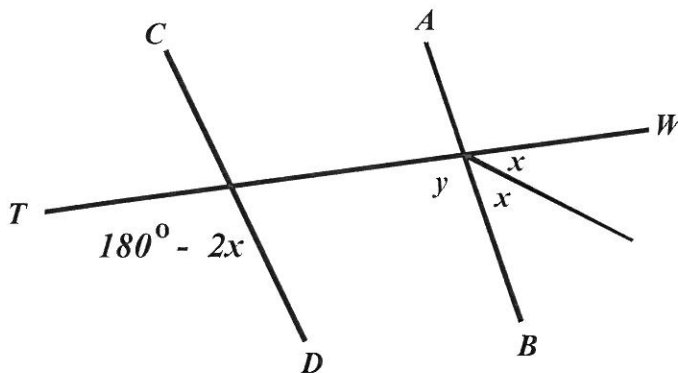
$$\therefore \widehat{T} = 180^\circ - 124^\circ = 56^\circ$$

(5) Write three conditions under which two lines will be parallel to one another.

Two lines will be parallel if:
* one pair of alternate angles are equal
or * one pair of corresponding angles are equal
or * one pair of co-interior angles are supplementary.

(6) Prove that $AB \parallel CD$ in each of the following sketches. Show all calculations:

(a)



$$y + x + x = 180^\circ \quad [\angle \text{ on str. line }]$$

$$\therefore y = 180^\circ - x - x$$

$$\therefore y = 180^\circ - 2x$$

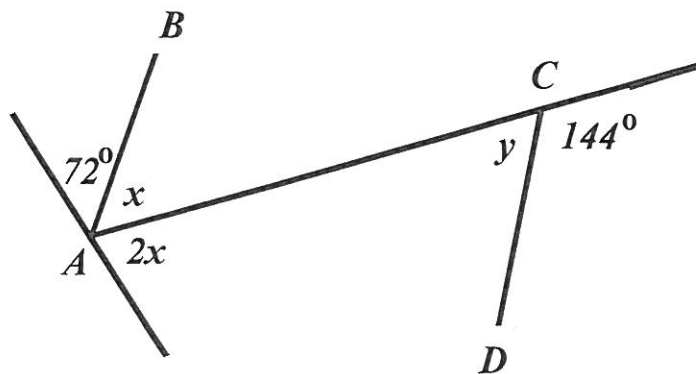
$$\text{but } \angle TSD = 180^\circ - 2x$$

$$\therefore \angle TSD = \angle S\hat{V}B = 180^\circ - 2x$$

but these are corresponding angles

$$\therefore AB \parallel CD$$

(b)



$$72^\circ + x + 2x = 180^\circ \quad [\text{L}^s \text{ on str. line}]$$

$$\therefore 72^\circ + 3x = 180^\circ$$

$$\therefore 3x = 180^\circ - 72^\circ = 108^\circ$$

$$\therefore x = \frac{108^\circ}{3} = 36^\circ$$

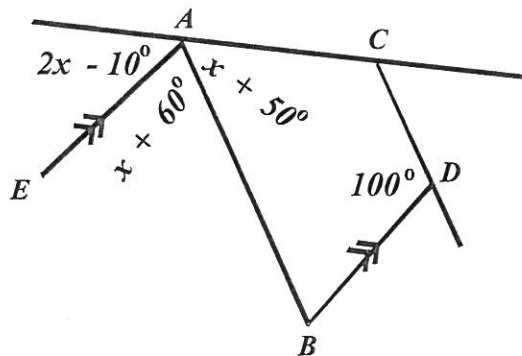
$$\text{but } y + 144^\circ = 180^\circ \quad [\text{L}^s \text{ on str. line}]$$

$$\therefore y = 180^\circ - 144^\circ = 36^\circ$$

$$\therefore y = x = 36^\circ$$

but these are alternate angles
 $\therefore AB \parallel CD$

(c)



$$2x - 10^\circ + x + 60^\circ + x + 50^\circ = 180^\circ \quad [\text{L}^s \text{ on str. line}]$$

$$\therefore 4x + 100^\circ = 180^\circ$$

$$\therefore 4x = 180^\circ - 100^\circ = 80^\circ$$

$$\therefore x = \frac{80^\circ}{4} = 20^\circ$$

$$\therefore \hat{EAB} = x + 60^\circ = 20^\circ + 60^\circ = 80^\circ$$

$$\text{but } \hat{B} = \hat{EAB} = 80^\circ \quad [\text{alt. L}^s; EA \parallel BD]$$

$$\text{but } \hat{B} + \hat{BDC} = 80^\circ + 100^\circ = 180^\circ$$

but these are co-interior angles

$$\therefore AB \parallel CD$$