Worksheet 2015-10ŁL-00006(1)

(1) Calculate the mid-point of each of the following lines through the given points:

(a) A(2 ; 6) and B(4 ; 4)

$MP_{AB} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{2+4}{2} ; \frac{6+4}{2}\right)$

$= \left(\frac{6}{2} ; \frac{10}{2}\right)$

$= (3 ; 5)$

(b) C(-2 ; 0) and D(-2 ; 2)

$MP_{CD} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-2-2}{2} ; \frac{0+2}{2}\right)$

$= \left(\frac{-4}{2} ; \frac{2}{2}\right)$

$= (-2 ; 1)$

(c) E(-1 ; 3) and F(2 ; -3)

$MP_{EF} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-1+2}{2} ; \frac{3-3}{2}\right)$

$= \left(\frac{1}{2} ; \frac{0}{2}\right)$

$= \left(\frac{1}{2} ; 0\right)$

(d) G(0 ; -6) and H(7 ; 1)

$MP_{GH} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{0+7}{2} ; \frac{-6+1}{2}\right)$

$= \left(\frac{7}{2} ; \frac{-5}{2}\right)$

(e) I(-7 ; -7) and J(1 ; 1)

$MP_{IJ} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-7+1}{2} ; \frac{-7+1}{2}\right)$

$= \left(\frac{-6}{2} ; \frac{-6}{2}\right)$

$= (-3 ; -3)$

(f) K(5 ; 1) and L(11 ; 1)

$MP_{KL} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5+11}{2} ; \frac{1+1}{2}\right)$

$= \left(\frac{16}{2} ; \frac{2}{2}\right)$

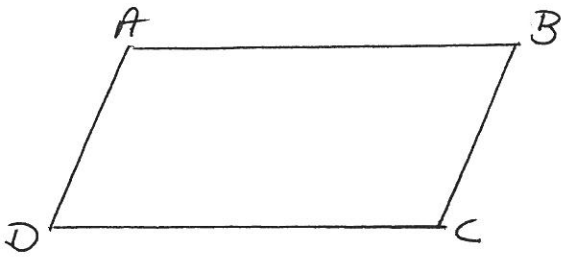
$= (8 ; 1)$

(2) A(-2 ; -1) , B(-1 ; -4) , C(3 ; -2) and D(2 ; 1) form the vertices of a quadrilateral.

(a) Determine the mid-points of the diagonals AC and BD.

(b) What do you notice from the answers in (a)?

(c) Which kind of quadrilateral is ABCD? Motivate.



(a) $A(-2;-1)$ and $C(3;-2)$

$$mp_{AC} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+3}{2} ; \frac{-1-2}{2}\right)$$

$$= \left(\frac{1}{2} ; \frac{-3}{2}\right)$$

$B(-1;-4)$ and $D(2;1)$

$$mp_{BD} = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-1+2}{2} ; \frac{-4+1}{2}\right)$$

$$= \left(\frac{1}{2} ; \frac{-3}{2}\right)$$

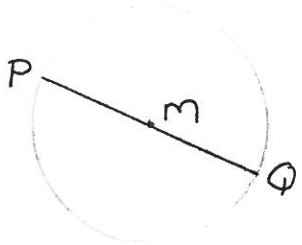
(b) Midpoints are the same.

(c) Parallelogram, because the diagonals bisect one another.

(3) P(-2 ; 5) and Q(6 ; 7) lie on the circumference of a circle. PQ is a diameter of the circle.

(a) Calculate the coordinates of the circle's centre.

(b) Calculate the length of the radius of the circle, rounded off to the nearest integer.



$\therefore$ M is the midpoint of PQ.

$\therefore M = \left(\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-2+6}{2} ; \frac{5+7}{2}\right)$

$= \left(\frac{4}{2} ; \frac{12}{2}\right)$

$M = (2;6)$

(b) $d(PM) = \sqrt{(x_P - x_M)^2 + (y_P - y_M)^2}$

$= \sqrt{(-2-2)^2 + (5-6)^2}$

$= \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{16+1}$

$= \sqrt{17}$

$\therefore d(PM) \approx 4$