



PREPARATORY EXAMINATION

2022

MARKING GUIDELINES

MATHEMATICS (PAPER 1) (10611)

23 pages

INSTRUCTIONS AND INFORMATION

A – Accuracy

C.A. – Consistent Accuracy

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- It is unacceptable for candidates to use adopted values/answers in solving questions.

QUESTION 1

1.1	1.1.1	$2x(x^2 - 1) = 0$ $2x(x - 1)(x + 1) = 0$ $x = 0$ or $x = 1$ or $x = -1$ NOTE: Any other valid method	✓ factors ✓ answers	(2)
	1.1.2	$x - 6 + \frac{2}{x} = 0$ $x^2 - 6x + 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$ $x = \frac{6 \pm \sqrt{28}}{2}$ $\therefore x = 5,65 \dots \text{or} \dots x = 0,35$ NOTE: Penalise for rounding-off in this question ONLY. Any other valid method	✓ standard form ✓ substitution into correct formula ✓✓ answers	(4)
	1.1.3	$(x - 1)(x + 4) \geq 6$ $\therefore x^2 + 3x - 4 \geq 6$ $\therefore x^2 + 3x - 10 \geq 0$ $\therefore (x + 5)(x - 2) \geq 0$ $\therefore x \leq -5$ OR $x \geq 2$ 	✓ standard form ✓ factors ✓ answers	(3)

1.2	$x - 2y = 1$ and $2x^2 - xy - 5y - 3y^2 - 2 = 0$ $x = 2y + 1 \quad \dots(1)$ $2x^2 - xy - 5y - 3y^2 - 2 = 0 \quad \dots(2)$ Substitute (1) into (2): $2(2y + 1)^2 - (2y + 1)y - 5y - 3y^2 - 2 = 0$ $3y^2 + 2y = 0$ $y(3y + 2) = 0$ $y = 0$ or $y = -\frac{2}{3}$ $x = 1$ or $x = -\frac{1}{3}$ NOTE: Any other valid method	✓ equation for x ✓ standard form ✓ y-values ✓ x-values	(4)
1.3	$2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x(2^1 + 1) = 3^y(3^2 - 1)$ $2^x(3) = 3^y(8)$ $\therefore \frac{2^x}{8} = \frac{3^y}{3}$ $\therefore \frac{2^x}{2^3} = \frac{3^y}{3}$ $\therefore 2^{x-3} = 3^{y-1}$ $\therefore x - 3 = 0$ and $y - 1 = 0$ $\therefore x = 3$ and $y = 1$ OR $2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x(2^1 + 1) = 3^y(3^2 - 1)$ $2^x(3) = 3^y(8)$ $2^x(3) = 3^y(2^3)$ $\therefore x = 3$ and $y = 1$	✓ factorise ✓ simplified equated bases ✓ answers ✓ factorise ✓ simplified equated bases ✓ answers	(3)

1.4	$x^2 + rx + m = 0$ and $x^2 + mx + r = 0$ $x^2 + rx + m = 0$ For real and equal roots, $\Delta = 0$ $b^2 - 4ac = 0$ $(r)^2 - 4(1)(m) = 0$ $r^2 - 4m = 0$ $r^2 = 4m$ $m = \frac{r^2}{4} \quad \dots (1)$ $x^2 + mx + r = 0$ $b^2 - 4ac = 0$ $m^2 - 4(1)(r) = 0$ $m^2 - 4r = 0 \quad \dots (2)$ Substitute (1) in (2) $\left(\frac{r^2}{4}\right)^2 - 4r = 0$ $\frac{r^4}{16} - 4r = 0$ $r^4 - 64r = 0$ $r(r^3 - 64) = 0$ $r(r - 4)(r^2 + 4r + 16) = 0$ $\therefore r = 4$ $m = \frac{r^2}{4}$ $m = \frac{4^2}{4}$ $\therefore m = 4$ OR	✓ substitute into $\Delta = 0$ ✓ equation for m ✓ equation 2 ✓ substitute for m ✓ value of r ✓ value of m OR	
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QUESTION 2

2.1	2.1.1	$ \begin{array}{cccc} 20 & 12 & 10 & 14 \\ \swarrow & & \swarrow & \swarrow \\ -8 & & -2 & 4 \\ & \swarrow & & \swarrow \\ & 6 & & 6 \end{array} $ $ \begin{array}{lll} 2a = 6 & 3a + b = -8 & a + b + c = 20 \\ a = 3 & 3(3) + b = -8 & 3 - 17 + c = 20 \\ & b = -17 & c = 34 \end{array} $ $T_n = 3n^2 - 17n + 34$	✓ 2nd diff. ✓ $a = 3$ ✓ $b = -17$ ✓ $c = 34$	(4)
	2.1.2	$T_n = a + (n-1)d$ $148 = -8 + (n-1)(6)$ $148 = 6n - 14$ $n = 27$ Between 27th and 28th terms.	✓ substitution into correct formula ✓ value for n ✓ conclusion	(3)
	2.1.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $\frac{n}{2}[2(-8) + (n-1)6] > 10140$ $3n^2 - 11n > 10140$ $3n^2 - 11n - 10140 > 0$ $(3n + 169)(n - 60) > 0$ $\therefore n > 60$ $n = 61$	✓ substitution ✓ standard form ✓ factors ✓ selection, $n > 60$ ✓ answer	(5)
2.2		$\sum_{r=1}^5 (r+b) = (1+b) + (2+b) + (3+b) + (4+b) + (5+b)$ $\sum_{r=1}^5 (r+b) = 15 + 5b$ $\therefore \sum_{r=1}^5 (r+b) = 10a$ $\therefore 15 + 5b = 10a$ $\therefore 5b = 10a - 15$ $\therefore b = 2a - 3$	✓ expansion ✓ equating ✓ answer	(3)

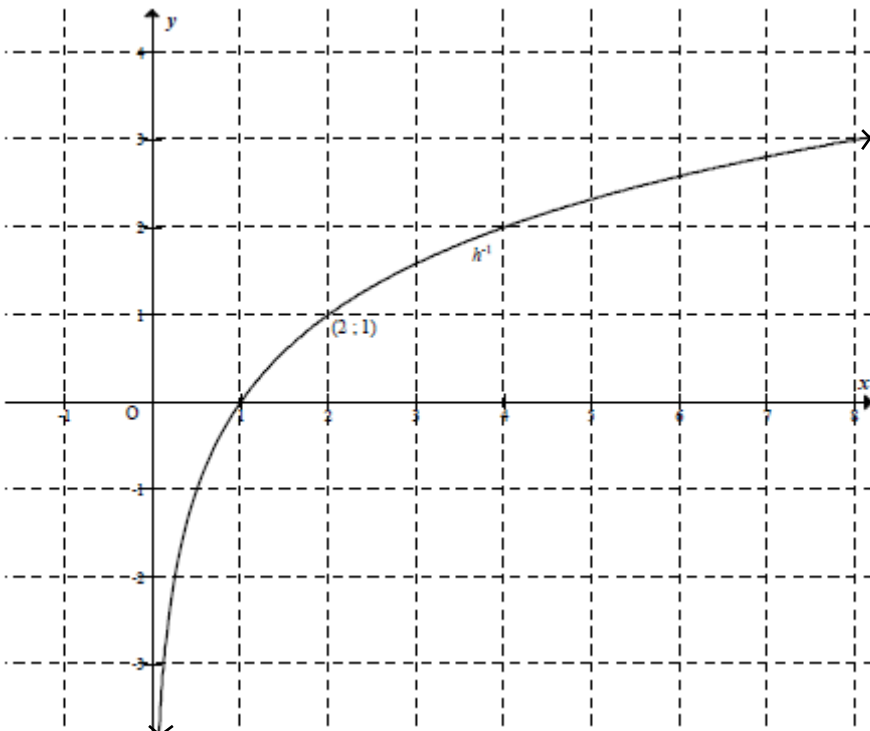
[15]

QUESTION 3

3.1	$a = \frac{24}{x}$ $r = \frac{6x}{12} \text{ or } \frac{3x^2}{6x} \text{ or } \frac{x}{2}$ $S_{\infty} = \frac{a}{1-r}$ $S_{\infty} = \frac{\frac{24}{x}}{1-\frac{x}{2}}$ $S_{\infty} = \frac{48}{2x-x^2}$	✓ value of r ✓ substitution in correct formula ✓ correct numerator ✓ correct denominator	(4)
3.2	It exists when: $-1 < \frac{x}{2} < 1 \text{ i.e.: } -1 < r < 1$ $\therefore -2 < x < 2$ ANSWER ONLY: Award full marks.	✓ $-1 < r < 1$ ✓ answer	(2)
3.3	$r > 1$ $\therefore \frac{x}{2} > 1$ $\therefore x > 2$ ANSWER ONLY: Award full marks.	✓ $r > 1$ ✓ answer	(2)
3.4	$a = \frac{24}{4}$ $\therefore a = 6$ $r = \frac{x}{2} = 2$ $n = 15$ $\therefore S_n = \frac{a(r^n - 1)}{r - 1}$ $\therefore S_{15} = \frac{6(2^{15} - 1)}{2 - 1}$ $\therefore S_{15} = 196\,602$	✓ value of a ✓ value of r ✓ substitution into correct formula ✓ answer	(4)
[12]			

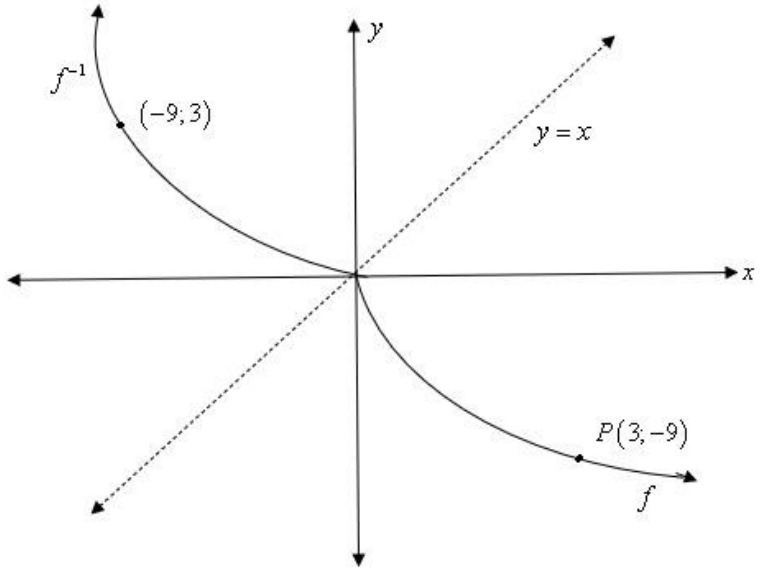
4.5	$k(x) = -g(x)$ $\therefore k(x) = -\left(\frac{2}{x-1} + 2\right)$ $\therefore k(x) = \frac{-2}{x-1} - 2$	✓ $k(x) = -g(x)$ ✓ answer	(2)
4.6	$x > 1$ or $0 < x < 1$ OR $x \in (1; \infty)$ or $(0 < x < 1)$ OR $x > 0$; $x \neq 1$ OR $x \in (0; \infty)$; $x \neq 1$	✓ answer ✓ answer ✓ answer ✓ answer ✓ answer ✓ answer ✓ answer ✓ answer	(2)
4.7	$k > 0$	✓ answer	(1)
[14]			

QUESTION 5

5.1	(0;1) NOTE: Answer MUST be in coordinate form.	✓ answer	(1)
5.2	$a^{-1} = \frac{1}{2}$ $\frac{1}{a} = \frac{1}{2}$ $a = 2$	✓ substitute point ✓ answer	(2)
5.3	$2^y = x$ $y = \log_2 x$ NOTE: Answer only, award FULL marks	✓ interchange x and y ✓ answer	(2)
5.4		✓ shape ✓ x-intercept	(2)
5.5	$x > 0$	✓ answer	(1)
5.6	$x > 2$	✓ answer	(1)

5.7	$g(x) = 100 \cdot 3^x \dots\dots h(x) = 2^x$ <i>if ... $g(x) = h(x)$</i> $\therefore 100 \cdot 3^x = 2^x$ $\therefore 100 = \frac{2^x}{3^x}$ $\therefore 100 = \left(\frac{2}{3}\right)^x$ $\therefore x = \log_{\frac{2}{3}} 100$ $\therefore x = -11,36$ OR $2 + x \log 3 = x \cdot \log 2$ $\therefore 2 = x \cdot \log 2 - x \log 3$ $\therefore 2 = x \cdot \log \frac{2}{3}$ $\therefore x = \frac{2}{\log \frac{2}{3}}$ $\therefore x = -11,36$	✓ simplification ✓ x as subject ✓ answer OR ✓ equating simplified sides ✓ x as subject ✓ answer	(3)
[12]			

QUESTION 6

6.1	$0 \leq x \leq 3$ NOTE: The answer may be written as separate inequalities. OR $x \in [0; 3]$	✓ critical values ✓ notation ✓ critical values ✓ notation	(2)
6.2	Equation of f : $y = -\sqrt{27x}$ for $x \geq 0$ Equation of f^{-1} : $x = -\sqrt{27y}$ for $y \geq 0$ $\therefore y = \frac{x^2}{27}$ for $x \leq 0$	✓ interchange x and y ✓ constraint of f^{-1} ✓ equation of y ✓ constraint of y	(4)
6.3		✓ shape ✓ x-intercept ✓ point $(-9 ; 3)$	(3)
6.4	Reflection in the x -axis	✓ answer	(1)
[10]			

QUESTION 7

NOTE: Incorrect formula, STOP marking, unless independent marks are being awarded.

7.1	$A = P(1+i)^n$ $2x = x(1 + \frac{i}{4})^{24}$ $\therefore (1 + \frac{i}{4})^{24} = 2$ $\therefore 1 + \frac{i}{4} = \sqrt[24]{2}$ $\therefore \frac{i}{4} = 0,0293$ $\therefore i = 0,1172$ The annual interest rate $\cong 11,72\%$ NOTE: If a candidate substitutes in their own values for A and P, award 4/5 marks.	✓ $n = 24$ ✓ substitution ✓ simplified sides ✓ value of i ✓ answer	(5)
7.2.1	$F_v = P_v (1 + i)^n$ $F_v = 10\,000 (1 + \frac{12}{100})^5$ $F_v = 10\,402,15$ NOTE: Any other valid method	✓ substitution ✓ answer	(2)
7.2.2	$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $\therefore 10\,402,15 = \frac{450[1 - (1,0079)^{-n}]}{0,0079}$ $\therefore [1 - (1,0079)^{-n}] = 0,183$ $\therefore 1 - 0,183 = (1,0079)^{-n}$ $\therefore 0,8169 = (1,0079)^{-n}$ $\therefore -n = \log_{1,0079} 0,8169$ $\therefore -n = -25,7008$ $\therefore n = 26 \text{ months}$ NOTE: Answer mark is for 26 months.	✓ substitution into the correct formula ✓ simplified sides ✓ correct use of logs ✓ answer	(4)

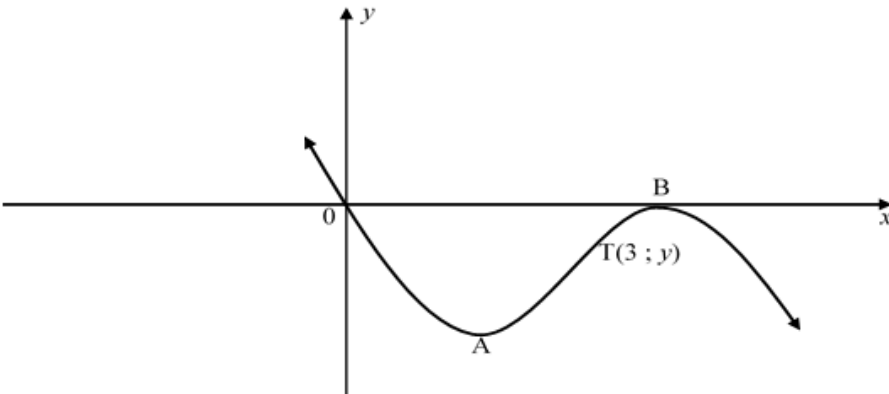
7.2.3	Balance of loan after 25th payment = value of loan – value of annuity at that time $= 10\,402,15(1,0079 \dots)^{25} - \frac{450 [(1,0079 \dots)^{25} - 1]}{0,0079}$ $= 12\,668,89 - 12\,386,53$ $= R282,36$ OR After the 25th payment, the remaining number of payments = $25,63 - 25 = 0,63$ $\therefore \text{balance} = \frac{450 [1 - (1 + \frac{0,095}{12})^{-0,70}]}{\frac{0,095}{12}}$ $\therefore \text{balance} = R282,36$	✓ substitution ✓ answer ✓ substitution ✓ answer	(2)
[13]			

QUESTION 8

8.1	$f(x) = 3x^2 + 2x$ $f(x+h) = 3(x+h)^2 + 2(x+h)$ $f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$ $f(x+h) - f(x) = 3x^2 + 6xh + 3h^2 + 2x + 2h - 3x^2 - 2x$ $= 6xh + 3h^2 + 2h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{h(6x + 3h + 2)}{h}$ $= \lim_{h \rightarrow 0} (6x + 3h + 2)$ $= 6x + 2$ NOTE: Penalise 1 mark for notation error in this question.	✓ $f(x+h)$ ✓ $f(x+h) - f(x)$ ✓ factorise ✓ answer	(4)
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8.2	$y = 4\sqrt{x} - \frac{8}{\sqrt{x}} + \pi x^3$ $y = 4x^{\frac{1}{2}} - 8x^{-\frac{1}{2}} + \pi x^3$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}} + 4x^{-\frac{3}{2}} + 3\pi x^2$ NOTE: Accept $\frac{2}{x^{\frac{1}{2}}} + \frac{4}{x^{\frac{3}{2}}} + 3\pi x^2$ OR $\frac{2}{\sqrt{x}} + \frac{4}{\sqrt{x^3}} + 3\pi x^2$	✓ change surds to rational exponents ✓✓✓ answers	(4)
8.3	This question has been removed from the question paper. Do not mark this question.		(0)
			[8]

QUESTION 9

			
9.1	$h(x) = -[x(x-6)^2]$ $h(x) = -[x^3 - 12x^2 + 36x]$ $h(x) = -x^3 + 12x^2 - 36x$ $\therefore a = 12$ $\therefore b = -36$	✓ substitute roots ✓ simplification ✓ simplification	(3)
9.2	$h(x) = -x^3 + 12x^2 - 36x$ $h'(x) = -3x^2 + 24x - 36$ $-3x^2 + 24x - 36 = 0$ $-3(x-6)(x-2) = 0$ $\therefore x = 6 \text{ or } x = 2$ $\therefore x = 2 \text{ at point A}$ $\therefore h(2) = -32$ $\therefore A(2; -32)$ NOTE: Answer does NOT have to be in coordinate form. Equating to zero must NOT be implied.	✓ derivative = 0 ✓ choice of x-value ✓ y-value	(3)
9.3	$h(3) = -(3)^3 + 12(3)^2 - 36(3)$ $y = -27$ Answer only, FULL marks	✓ answer	(1)

9.4	$h(x) = -x^3 + 12x^2 - 36x$ $h'(x) = -3x^2 + 24x - 36$ $h''(x) = -6x + 24$ $h''(3) = -6(3) + 24$ $h''(3) = 6$ $\therefore h''(3) > 0$ $\therefore h$ is concave UP at point T. OR $h''(x) = 0$ $-6x + 24 = 0$ $x = 4$ Point of Inflection is at $x = 4$ at Point T, the graph is CONCAVE UP NOTE: Candidate must have a valid CALCULATION to reach a conclusion for full marks. If only the conclusion is given, award 1 mark.	✓ $h''(x)$ ✓ value of $h''(3)$ ✓ conclusion ✓ $h''(x)$ ✓ point of inflection at $x = 4$ ✓ conclusion	(3)
9.5	$h''(x) = -6x + 24$ $\therefore 0 = -6x + 24$ $\therefore 6x = 24$ $\therefore x = 4$ $\therefore h(4) = -16$ Point of inflection is: (4 ; -16) NOTE: Answer does not have to be in coordinate form.	✓ value of x ✓ value of y	(2)
			[12]

QUESTION 10

10.1	Rent R500 R600 Occupation (Rented) 72 72 – 2 Income R36 000 Number of increases: $\frac{x-500}{100}$ Occupation (Rented): $72 - 2(\frac{x-500}{100})$ Income(I) = $x[72 - \frac{2x}{100} + \frac{1\,000}{100}]$ $= x[\frac{7\,200 - 2x + 1\,000}{100}]$ $= x[\frac{8\,200 - 2x}{100}]$ $= 82x - \frac{x^2}{50}$	✓ $\frac{x-500}{100}$ ✓ $72 - 2(\frac{x-500}{100})$ ✓ substitution ✓ $x[\frac{7\,200 - 2x + 1\,000}{100}]$ ✓ $= x[\frac{8\,200 - 2x}{100}]$	(5)
10.2	$I = 82x - \frac{x^2}{50}$ $I' = 82 - \frac{2x}{50}$ $82 - \frac{2x}{50} = 0$ $82 = \frac{2x}{50}$ $x = R2\,050$ NOTE: Equating to zero MAY be implied.	✓ derivative = 0 ✓ answer	(2)
[7]			

QUESTION 11

11.1		$P(A \text{ or } B) = P(A) + P(B)$ $0,57 = P(A) + 2 P(A)$ $\therefore 3P(A) = 0,57$ $\therefore P(A) = 0,19$ $\therefore P(B) = 2 P(A)$ $= 2(0,19)$ $P(B) = 0,38$	✓ substitution ✓ value of P(A) ✓ answer	(3)
11.2	11.2.1	Not Defective (ND) $\therefore P(ND) = \frac{35}{40} \text{ or } \frac{7}{8} \text{ or } 0,88$	✓ answer	(1)
	11.2.2	Not Defective (ND)/Defective (D) $P(ND \text{ and } D) + P(D \text{ and } ND)$ $= \left(\frac{35}{40} \cdot \frac{5}{39}\right) + \left(\frac{5}{40} \cdot \frac{35}{39}\right)$ $= \frac{35}{156} \text{ or } 0,22$	✓ formula (may be implied) ✓ substitution ✓ answer	(3)
	11.2.3	Defective (D) $P(D \text{ and } D)$ $= \frac{5}{40} \cdot \frac{4}{39}$ $= \frac{20}{1560} \text{ or } \frac{1}{78} \text{ or } 0,01$	✓ $\frac{5}{40}$ ✓ $\frac{4}{39}$ ✓ answer	(3)

11.3	11.3.1	$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ OR $7!$ OR 5 040	✓ answer	(1)
	11.3.2	Total number of arrangements = $2! \times 6!$ OR 1 440	✓ answer	(1)
	11.3.3	Economics books = $4!$ Life Sciences books = $3!$ Economics and Life Sciences = $2!$ Total number = $4! \times 3! \times 2!$ OR $24 \times 6 \times 2$ OR 288	✓ combinations for all criteria ✓ answer	(2)
				[14]
TOTAL:				144

Note: The mark out of 144 MUST be converted to 150.

DETAILED EXPLANATION OF QUESTION 10

- For every R100 that the price of a room increases, 2 rooms less are let.
- To determine the amount of rooms that been let, the number of increases of R100 must be determined because that value must be multiplied by 2 to subtract from 72.
- Example: If the rooms are let for R800 per day, then the number of increases of R100 is 3.

Therefore: Number of increases = $\frac{800 - 500}{100}$

Therefore, $72 - 2(3) = 66$ rooms let.

Income will be: $R800 \times 66 = R52\ 800$.

Now, the new price is not R800, but Rx , making the formula:

$$\text{Number of increases} = \frac{x - 500}{100}$$

Therefore, the number of rooms let = $72 - 2\left(\frac{x - 500}{100}\right)$

Therefore, the income = $x \left[72 - 2\left(\frac{x - 500}{100}\right)\right]$