MEMO 2015-9ƎC-00001(1)

(1) Write the next five numbers in each sequence:
Also write the pattern in words:

(a) $-3\ ;\ -11\ ;\ -19\ ;\ -27\ ;$ **$-35\ ;\ -43\ ;\ -51\ ;\ -59\ ;\ -67$**

Subtract 8.

$\therefore\ -8$

(b) $5\ ;\ 8\ ;\ 11\ ;\ 14\ ;$ **$17\ ;20\ ;23\ ;26\ ;29$**

Add 3.

$\therefore\ +3$

(c) $98\,415\ ;\ 32\,805\ ;\ 10\,935\ ;\ 3\,645\ ;$ **$1\,215\ ;405\ ;135\ ;45\ ;15$**

Divide by 3.

$\therefore \div 3$

(d) $1\ ;\ 1\ ;\ 3\ ;\ 4\ ;\ 5\ ;\ 9\ ;\ 7\ ;\ 16\ ;\ 9\ ;$ **$25\ ;11\ ;36\ ;13\ ;49$**

Blue pattern: Add 2 → +2 or odd natural numbers

Green pattern: Complete squares

(2) The general term for a sequence is $T_n = 5n + 3$.

(a) Determine the first four terms of the sequence.
(b) Determine the 85^{th} term of the sequence.
(c) Which term in the sequence is equal to 523?

(a) $\mathbf{T_1 = 5(1) + 3 = 5 + 3 = 8}$
$\mathbf{T_2 = 5(2) + 3 = 10 + 3 = 13}$
$\mathbf{T_3 = 5(3) + 3 = 15 + 3 = 18}$
$\mathbf{T_4 = 5(4) + 3 = 20 + 3 = 23}$
$\therefore$ The first four terms of the sequence: 8 ; 13 ; 18 ; 23

(b) $\mathbf{T_n = 5n + 3}$
$\therefore\ \mathbf{T_{85} = 5(85) + 3 = 425 = 3}$
$\therefore\ \mathbf{T_{85} = 428}$

(c) $T_n = 5n + 3$

$\therefore\ 523 = 5n + 3$

$\therefore\ 523 - 3 = 5n$

$\therefore\ 520 = 5n$

$\therefore\ \frac{520}{5} = n$

$\therefore\ n = 104$

(3) Complete the following table and answer the questions:

(a)

Position in sequence	1	2	3	7	11	*28	35
Term	−4	−7	−10	−22	−34	−85	−106

(b) Describe the pattern in words.

(c) Determine the general term of the second difference of the above sequence as T_n = ...

(d) Determine the 100th term of the sequence.

(b) Subtract 3 → −3

(c) $T_0 = -1$ → **Add 3 to the −4 (the first term of the sequence)**

$\therefore\ T_n = -3n - 1$

→ **The −3 is number that is "added". This number that is added can also be negative as in this case.**

→ **The −1 is obtained by calculating "T_0".**

Or the formula can be used:

$T_n = a + (n-1)d$ **with** a **the first term and** d **the number to be "added".**

$\therefore\ T_n = -4 + (n-1)(-3)$

$\therefore\ T_n = -4 - 3n + 3$

$\therefore\ T_n = -1 - 3n = -3n - 1$

(d) $T_n = -3n - 1$

$\therefore\ T_{100} = -3(100) - 1$

$\therefore\ T_{100} = -300 - 1$

$\therefore\ T_{100} = -301$

* **Can also be determined by using the general term**

$T_n = -85 = -3n - 1$

$\therefore\ 3n = 85 - 1 = 84$

$\therefore\ 3n = \frac{84}{3} = 28$

(4) Fill in the missing numbers for each sequence and determine the general term (T_n):

(a) **3** ; 6 ; 12 ; 24 ; **48** ; **96** ; **192** ; 384 $\rightarrow \times 2$

$T_1 = 3 = 3 \times 1 = 3 \times 2^0$

$T_2 = 6 = 3 \times 2 = 3 \times 2^1$

$T_3 = 12 = 3 \times 2 \times 2 = 3 \times 2^2$

$T_4 = 24 = 3 \times 2 \times 2 \times 2 = 3 \times 2^3$

$\therefore\ T_n = 3 \times 2^{n-1}$

(b) $8\frac{1}{2}$; $7\frac{3}{4}$; **7** ; $6\frac{1}{4}$; $5\frac{1}{2}$; $4\frac{3}{4}$; 4 $\rightarrow -\frac{3}{4}$

$T_0 = 8\frac{1}{2} + \frac{3}{4} = 8\frac{2}{4} + \frac{3}{4} = 8\frac{5}{4} = 9\frac{1}{4}$

$\therefore\ T_n = -\frac{3}{4}n + 9\frac{1}{4}$

or $T_n = a + (n-1)d$

$\therefore\ T_n = 8\frac{1}{2} + (n-1)\left(-\frac{3}{4}\right)$

$\therefore\ T_n = 8\frac{1}{2} - \frac{3}{4}n + \frac{3}{4}$

$\therefore\ T_n = -\frac{3}{4}n + 9\frac{1}{4}$

(c) 136 ; **139** ; 142 ; **145** ; 148 ; **151** ; 154 ; **157** $\rightarrow +3$

$T_0 = 136 - 3 = 133$

$\therefore\ T_n = 3n + 133$

or $T_n = a + (n-1)d$

$\therefore\ T_n = 136 + (n-1)(3)$

$\therefore\ T_n = 136 + 3n - 3$

$\therefore\ T_n = 3n + 133$