



**Wes-Kaapse
Regering**

Onderwys

OVERBERG EDUCATION DISTRICT

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2016

MEMORANDUM

MARKS: 150

TIME: 3 hours

This paper consist of 12 pages and a formula sheet

QUESTION 1																		
1.1	Skew to the right/ positively skewed <i>Skeef na regs/ positief skeef</i>					✓ right/ positively regs/ <i>positief</i>			(1)									
1.2	70 - 30 = 40					✓ 70 - 30 ✓ Answer/ <i>Antw</i> Answer only: Full marks			(2)									
1.3	75%					✓ Answer/ <i>Antw</i>			(2)									
1.4	<table border="1"><tr><td>20</td><td>30</td><td>30</td><td>33</td><td>40</td><td>58</td><td>66</td><td>74</td><td>90</td></tr></table> $T_4: \frac{20+2(30)+T_4+40+58+66+74+90}{9} = 49$ $\therefore 408 + T_4 = 441 \Rightarrow T_4 = 33$					20	30	30	33	40	58	66	74	90	✓ 20 ✓ 30 ✓ 30 ✓✓ 33 ✓ 40 ✓ 90			(7)
20	30	30	33	40	58	66	74	90										
									[12]									

QUESTION 2			
2.1		✓✓ Plot values / Plot waardes(-1 per mistake/fout) ✓ Anchored at (0;0) / Geanker by (0;0) ✓ Connect points / Verbind punte	(4)
2.2	$40 \leq t < 60$	✓ answer/antwoord	(1)
2.3	172	✓ answer/antwoord	(1)
2.4	(72;148) $\therefore 172-148 = 24$ learners/leerders	✓ 148(accept/aanvaar 146-150) ✓ 24(accept/aanvaar 22-26)	(2)
2.5	Frequency/Frekwensie: 30; 39; 60;28;10;5 $\frac{30 \times 10 + 39 \times 30 + 60 \times 50 + 28 \times 70 + 10 \times 90 + 5 \times 110}{172}$ $= \frac{7880}{172}$ $= 45,81$	✓ frequency/frekwensie ✓ midpointsmiddelpunte ✓ $\frac{7880}{172}$ ✓ answer/antwoord	(4)
			[12]

QUESTION 3			
3.1.1	$m_{AC} = -\frac{1}{3}$ $m_{BD} = 3$ [Diagonals of a rhombus]/ [Hoeklyne van ruit/rombus] $y - y_1 = m(x - x_1)$ $y - 9 = 3(x - 3)$ $3x - y = 0$	✓ m_{AC} ✓ m_{BD} ✓ subst, $m = 3$ & $(3;9)$ in eqn. / verv. $m = 3$ & $(3;9)$ in verg.	(3)
3.1.2	$x + 3y = 10 \dots \dots \dots (1)$ $3x - y = 0 \dots \dots \dots (2)$ $3x + 9y = 30 \dots \dots \dots (3) \quad (1) \times 3$ $10y = 30 \quad (3) - (2)$ $y = 3$ $x + 3(3) = 10$ $x = 1$ K(1; 3)	✓ equating two eqns / gelyk stel van twee vergs. ✓ simplification / vereenvoudiging ✓ $y = 3$ ✓ $x = 1$	(4)
3.1.3	$\left(\frac{x+3}{2}; \frac{y+9}{2}\right) = (1; 3)$ $\frac{x+3}{2} = 1 \quad \frac{y+9}{2} = 3$ $x+3 = 2 \quad \& \quad y+9 = 6$ $x = -1 \quad \& \quad y = -3$ B(-1; -3)	✓ method using midpoint / metode gebruik middelpunt ✓ x ✓ y	(3)
3.1.4	$\sqrt{(3-5)^2 + (9-y)^2} = \sqrt{(-1-5)^2 + (y+3)^2}$ $4 + 81 - 18y + y^2 = 36 + y^2 + 6y + 9$ $-24y = -40$ $y = \frac{5}{3}$	✓ $CD=BC$ ✓ subst. in afstandsformule ✓ simplify/vereenv ✓ ans/antw	(4)
3.2.1	$m_{PQ} = \frac{8-2}{5-(-3)} = \frac{6}{8}$ $= \frac{3}{4}$	✓ subs into eqn / verv. in verg ✓ answer/antwoord	(2)
3.2.2	$\tan \theta = \frac{3}{4} \quad \theta = 36,9$	✓ $\tan \theta$ ✓ answer /antwoord	(2)
3.2.3	$y = \frac{3}{4}x + c$ $0 = \frac{3}{4}(8) + c$ $c = -6$ $y = \frac{3}{4}x - 6$	✓ subst/verv. $m = \frac{3}{4}$ ✓ subst/verv. (8:0) ✓ answer/antwoord	(3)
			[21]

QUESTION 4			
4.1	$C(-2; -3)$	✓ -2 ✓ -3	(2)
4.2	$OB = \sqrt{(-6)^2 + (-4)^2} = \sqrt{52} = 2\sqrt{13}$ $\therefore \text{radius} = \sqrt{13} \text{ units/ eenhede}$ <i>OF/OR</i> $BC = \sqrt{(-4 - 2)^2 + (-6 + 3)^2}$ $= \sqrt{13}$	✓ subst in formule ✓ Answer/ Antw. ✓ subst in formule ✓ radius	(2)
4.3	$\therefore (x + 2)^2 + (y + 3)^2 = 13$	✓ $(x + 2)^2 + (y + 3)^2$ ✓ 13	(2)
4.4	Radius/ diameter $\perp$ tangent/raaklyn	✓ Reason/ Rede	(1)
4.5	$m_{OB} = \frac{-6}{-4} = \frac{3}{2}$ $\therefore m_{KL} = -\frac{2}{3}$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y + 6 = -\frac{2}{3}(x + 4)$ c $y = -\frac{2}{3}x - 8\frac{2}{3}$ OR $y = mx + c$ $-6 = -\frac{2}{3}(-4) + c$ $-8\frac{2}{3} = c$	✓ $m_{OB} = \frac{3}{2}$ ✓ $m_{KL} = -\frac{2}{3}$ ✓ Substitute/ Vervang m and/ en pt(-4; -6) ✓ Equation/ Vergelyking	(4)
4.6	Let/ Stel $x = 0$ $\therefore (2)^2 + (y + 3)^2 = 13$ $\therefore 4 + y^2 + 6y + 9 - 13 = 0$ $\therefore y^2 + 6y = 0$ $\therefore y(y + 6) = 0$ $y = 0 \text{ or/ of } y = -6$ $\Rightarrow E(0; -6)$ OR $BE \perp OE \quad \angle \text{ in semi circle/ } \angle \text{ in halwe sirkel}$ $\therefore BE \parallel x\text{-axis/ as}$ $\Rightarrow E(0; -6)$	✓ $x = 0$ ✓ Standard form/ Standaardvorm ✓ factors/ faktore ✓ $E(0; -6)$ ✓ S ✓ R ✓ $BE \parallel x\text{-axis/ as}$ ✓ $E(0; -6)$	(4)
4.7	$\frac{\text{oppervlakte } \triangle OBE}{\text{oppervlakte } \triangle OBL} = \frac{\frac{1}{2} OE \times BE}{\frac{1}{2} OL \times BE}$ $= \frac{OE}{OL}$ $= \frac{6}{8\frac{2}{3}}$ $= \frac{9}{13}$	✓ $\frac{\frac{1}{2} OE \times BE}{\frac{1}{2} OL \times BE}$ ✓ $\frac{OE}{OL}$ ✓ $\frac{6}{8\frac{2}{3}}$ ✓ Answer/ Antw.	(4)
			[19]

QUESTION 5			
5.1	$1 - \sin^2 \theta + 3 - \cos^2 \theta$ $= \cos^2 \theta + 3 - \cos^2 \theta$ $= 3$	✓ subst $\cos^2 \theta$ / vervang $\cos^2 \theta$ ✓✓ answer/antwoord	(3)
5.2	$\frac{\cos 160^\circ \tan 200^\circ}{2 \sin(-10^\circ)}$ $= \frac{-\cos 20^\circ \tan 20^\circ}{-2 \sin 10^\circ}$ $= \frac{-\cos 20^\circ \frac{\sin 20^\circ}{\cos 20^\circ}}{-2 \sin 10^\circ}$ $= \frac{\sin 20^\circ}{2 \sin 10^\circ}$ $= \frac{2 \sin 10^\circ \cos 10^\circ}{2 \sin 10^\circ}$ $= \cos 10^\circ$	✓ $-\cos 20^\circ$ ✓ $\tan 20^\circ$ ✓ $-2 \sin 10^\circ$ ✓ $\frac{\sin 20^\circ}{\cos 20^\circ}$ ✓ $2 \sin 10^\circ \cos 10^\circ$ ✓ $\cos 10^\circ$	(6)
5.3	$LHS : \frac{\cos^2 x \sin^2 x + \cos^4 x}{1 - \sin x}$ $= \frac{\cos^2 x (\sin^2 x + \cos^2 x)}{1 - \sin x}$ $= \frac{1 - \sin^2 x (1)}{1 - \sin x}$ $= \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)}$ $= 1 + \sin x = RHS$	✓ common factor / gemene factor ✓ $1 - \sin^2 x$ ✓ 1 ✓ factorise / faktorisier	(4)
5.4.1	$\cos(x + 45^\circ) \cos(x - 45^\circ) = \frac{1}{2} \cos 2x$ $= (\cos x \cos 45^\circ - \sin x \sin 45^\circ)(\cos x \cos 45^\circ + \sin x \sin 45^\circ)$ $= \cos^2 x \cos^2 45^\circ - \sin^2 x \sin^2 45^\circ$ $= \cos^2 x \left(\frac{\sqrt{2}}{2}\right)^2 - \sin^2 x \left(\frac{\sqrt{2}}{2}\right)^2$ $= \left(\frac{\sqrt{2}}{2}\right)^2 (\cos^2 x - \sin^2 x)$ $= \left(\frac{\sqrt{2}}{2}\right)^2 \cos 2x$ $= \frac{1}{2} \cos 2x$	✓ Expanding / Uitbreiding ✓ Simplify / Vereenvoudig ✓ Special triangles / Spesiale driehoek ✓ Double angle / Dubbelhoek	(4)

5.4.2	$\frac{1}{2}\cos 2x = -\frac{1}{2}$ $\Rightarrow \cos 2x = -1$ $2x = 0^\circ$ $2x = 180^\circ + 360k$ $\Rightarrow x = 90^\circ - 180k$ $\therefore x = 90^\circ$	$\checkmark \min -\frac{1}{2}$ $\checkmark 2x = 180^\circ$ $\checkmark x = 90^\circ$	(3)
			[20]

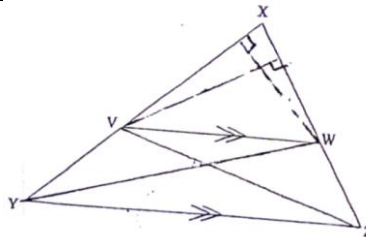
QUESTION 6			
6.1	120°	$\checkmark$ Answer/ Antw	(1)
6.2	1	$\checkmark$ Answer/ Antw	(1)
6.3	$-2 \leq y \leq 0$ Or/ Of $y \in [-2; 0]$	$\checkmark$ Answer/Antw	(1)
6.4	$\cos(x - 60^\circ) = \sin 3x$ $= \cos(90^\circ - 3x^\circ)$ $x - 60^\circ = 90^\circ - 3x + n \cdot 360^\circ \text{ or } x - 60^\circ = 360^\circ - (90^\circ - 3x) + n \cdot 360^\circ$ $4x = 150 + n \cdot 360^\circ \text{ or } -2x = 330^\circ + n \cdot 360^\circ$ $x = 37,5^\circ + n \cdot 90^\circ \text{ or }$ $x = -165^\circ - n \cdot 180^\circ, n \in \mathbb{Z}$ $\therefore x = 37,5^\circ; 15^\circ; -52,5^\circ$ OR/OR $\cos(x - 60^\circ) = \sin 3x$ $= \cos(90^\circ - 3x^\circ)$ $90^\circ - 3x = x - 60^\circ + n \cdot 360^\circ \text{ or } 90^\circ - 3x = 360^\circ - (x - 60^\circ) + n \cdot 360^\circ$ $-4x = -150 + n \cdot 360^\circ$ $2x = 330^\circ + n \cdot 360^\circ$ $x = 37,5^\circ - n \cdot 90^\circ \text{ or }$ $x = 165^\circ + n \cdot 180^\circ, n \in \mathbb{Z}$ $\therefore x = 37,5^\circ; 15^\circ; -52,5^\circ$ OR/OF $\cos(x - 60^\circ) = \sin 3x$ $\therefore \sin[90^\circ - (x - 60^\circ)] = \sin 3x$	$\checkmark$ Equating/ Vergelyking $\checkmark \sin 3x = \cos(90^\circ - 3x^\circ)$ $\checkmark$ both equations/ beide vergelykings: $x - 60^\circ = 90^\circ - 3x + n \cdot 360^\circ$ or/ of $x - 60^\circ = 360^\circ - (90^\circ - 3x) + n \cdot 360^\circ$ $\checkmark \checkmark$ both general solutions/ beide algemene oplossings $\checkmark 37,5^\circ \& 15^\circ \& -52,5^\circ$ (-1 per mistake) OR/OF $\checkmark$ Equating/ Vergelyking $\checkmark \sin 3x = \cos(90^\circ - 3x)$ $\checkmark$ both equations/ beide vergelykings: $90^\circ - 3x = x - 60^\circ + n \cdot 360^\circ$ or $90^\circ - 3x = 360^\circ - (x - 60^\circ) + n \cdot 360^\circ$ $\checkmark \checkmark$ both general solutions/ beide algemene oplossings $\checkmark 37,5^\circ \& 15^\circ \& -52,5^\circ$ (-1 per mistake) OR/OF $\checkmark$ Equating/ Vergelyking $\checkmark \cos(x - 60^\circ) = \sin(150^\circ -$	(6)

	$\therefore \sin(150^\circ - x) = \sin 3x$ $150^\circ - x = 3x + n \cdot 360^\circ \quad \text{or}$ $(150^\circ - x) = 180^\circ - 3x + n \cdot 360^\circ$ $-4x = -150^\circ + n \cdot 360^\circ \text{ or}$ $2x = 30^\circ + n \cdot 360^\circ$ $x = 37,5^\circ - n \cdot 90^\circ \quad \text{or}$ $x = 15^\circ + n \cdot 180^\circ, \quad n \in \mathbb{Z}$ $\therefore x = 37,5^\circ; 15^\circ; -52,5^\circ$ OR/ OF $\sin[90^\circ - (x - 60^\circ)] = \sin 3x$ $\therefore \sin 3x = \sin(150^\circ - x)$ $3x = 150^\circ - x + n \cdot 360^\circ \quad \text{or} \quad 3x = 180^\circ - (150^\circ - x) + n \cdot 360^\circ$ $4x = 150^\circ + n \cdot 360^\circ$ $2x = 30^\circ + n \cdot 360^\circ$ $x = 37,5^\circ + n \cdot 90^\circ \text{ or}$ $x = 15^\circ + n \cdot 180^\circ, \quad n \in \mathbb{Z}$ $\therefore x = 37,5^\circ; 15^\circ; -52,5^\circ$	$x)$ ✓ both equations/ <i>beide vergelykings</i> : $150^\circ - x = 3x + n \cdot 360^\circ \quad \text{or}$ $(150^\circ - x) = 180^\circ - 3x + n \cdot 360^\circ$ ✓✓ both general solutions/ <i>beide algemene oplossings</i> ✓ $37,5^\circ$ & 15° & $-52,5^\circ$ (-1 per mistake) OR/OF ✓ Equating/ <i>Vergelyking</i> ✓ $\cos(x - 60^\circ) = \sin(150^\circ - x)$ ✓ both equations/ <i>beide vergelykings</i> : $3x = 150^\circ - x + n \cdot 360^\circ \quad \text{or}$ $3x = 180^\circ - (150^\circ - x) + n \cdot 360^\circ$ ✓✓ both general solutions/ <i>beide algemene oplossings</i> ✓ $37,5^\circ$ & 15° & $-52,5^\circ$ (-1 per mistake)	
6.5.1	$20^\circ < x < 42,5^\circ$ (graphs shift 5° to the right)	✓ 20° , ✓ $42,5^\circ$ ✓ Notasie	(3)
6.6	$\cos(x - 60^\circ + 45^\circ)$ $= \cos(x - 15^\circ)$	✓ 45° ✓ answer / antwoord	(2)
			[14]

QUESTION 7			
7.1	$\widehat{PBA} = \alpha$	✓ Answer/ Antw.	(1)
7.2	$\text{Area } \triangle ABC = t = \frac{1}{2} AB \cdot k \sin \alpha$ $2t = AB \cdot k \sin \alpha$ $AB = \frac{2t}{k \sin \alpha}$	✓ Area of/ van $\triangle ABC$ ✓ Simplify/ Vereenvoudig	(2)
7.3	$\frac{AB}{PB} = \cos \alpha$ $PB = \frac{AB}{\cos \alpha}$ $= \frac{2t}{k \sin \alpha \cos \alpha}$ $= \frac{4t}{2 k \sin \alpha \cos \alpha}$ $= \frac{4t}{k \sin 2\alpha}$	✓ correct cos ratio/ Korrekte cos-verh. ✓ PB subject/ onderwerp ✓ Substitute/ Vervang AB ✓ $PB = \frac{4t}{2 k \sin \alpha \cos \alpha}$	(4)
			[7]

QUESTION 8			
8.1	Supplementary / <i>Supplementêr</i> (saam 180°)	✓ answer/ antwoord	(1)
8.2.1	$\widehat{EFO} = 90^\circ$ [tan radius]/[raaklyn radius] $\widehat{EGO} = 90^\circ$ [tan radius]/[raaklyn radius] $\widehat{EFO} + \widehat{EGO} = 180^\circ$ $\therefore$ FOGE is cyclic quad [opp angles supplementary] <i>is 'n koordevierhoek [teenoorst. hoeke supplementêr.]</i>	✓ S ✓ R ✓ S ✓ R ✓ R	(5)
8.2.2	$\widehat{G}_1 = \widehat{H} = x$ [tan chord]/[raaklyn koord] $\widehat{R}_1 = \widehat{H} = x$ [corresp angles/ooreenkmst hoeke; $EK \parallel FH$] $\therefore \widehat{G}_1 = \widehat{R}_1$ EG is a tangent / is 'n raaklyn [angle between line and chord] / [hoek tussen lyn en koord]	✓ S ✓ R ✓ S ✓ R ✓ R	(5)
8.2.3	$\widehat{O}_1 = 2\widehat{H}$ [angle at centre]/[middelpuntshoek] $= 2x$ $\therefore \widehat{FEG} = 180^\circ - 2x$ [opp angles of cyclic quad] / <i>[teenoorst. hoeke van koordevierhoek]</i>	✓ S ✓ R ✓ R	(3)
			[14]

QUESTION 9			
9.1.1	$\widehat{BAD} = 90^\circ \dots \angle$ in semi circle/ $\angle$ in halwe sirkel	✓ S ✓R	(2)
9.1.2	$\widehat{AOD} = 76^\circ \dots \angle$ at centre = $2 \times \angle$ at circumf./ midpts $\angle = 2 \times$ omtreks $\angle$	✓ S ✓R	(2)
9.1.3	$\widehat{CDE} = 63^\circ \dots$ ext $\angle$ of cyclic quad/ buite $\angle$ van koordevierhoek	✓ S ✓R	(2)
9.1.4	$\widehat{DCE} = 25^\circ \dots$ tan-chord thm/ raakl- koordstelling	✓ S ✓R	(2)
9.1.5	$\widehat{ACD} = 38^\circ \dots \angle$ at centre = $2 \times \angle$ at circumf. OR $\angle$ s in same / Segment / midpts $\angle = 2 \times$ omtreks $\angle$ OF $\angle$ e in dies. segm	✓ S ✓R	(2)
			[10]

QUESTION 10			
10.1	 Construction / Konstruksie: VZ, WY $\frac{\text{area DXVW}}{\text{area DVWY}} = \frac{\frac{1}{2} XV \cdot \text{hoogte lyn}}{\frac{1}{2} VY \cdot \text{hoogte lyn}}$ $= \frac{XV}{VY}$ $\frac{\text{area DXVW}}{\text{area DWVZ}} = \frac{\frac{1}{2} XW \cdot \text{hoogte lyn}}{\frac{1}{2} WZ \cdot \text{hoogte lyn}}$ $= \frac{XW}{WZ}$ Area DVWY = Area DWVZ Dieselfde basis VW en gelyke hoogtes tussen VW // ZY $\therefore \frac{XV}{VY} = \frac{XW}{WZ}$	✓ Construction / Konstruksie Dieselfde hoogte lyn ✓ Area Dieselfde hoogte lyn ✓ Area ✓ area equal / area gelyk VW//ZY ✓	(6)

10.2.1	$PA: AQ = 3:5$ $\frac{\text{area} \Delta PRA}{\text{area} \Delta QRA} = \frac{\frac{1}{2} \times 3 \times \text{hoogte}}{\frac{1}{2} \times 5 \times \text{hoogte}}$ $= \frac{PA}{AQ} = \frac{3}{5}$	✓ areas ✓ answer / antwoord	(2)
10.2.2	$\frac{PC}{CA} = \frac{PB}{BR} = \frac{1}{2}$ (line parallel to one side of Δ) $\frac{BD}{BQ} = \frac{CA}{CQ} = \frac{2}{7}$ (lyn parallel aan een sy van Δ)	✓ $\frac{1}{2}$ ✓ R ✓ $\frac{2}{7}$ ✓ R	(4)
			[12]
QUESTION 11			
11.1	$\angle D_2 = 90^\circ$ [line from centre to midpoint of chord]/[lyn vanaf die middelpnt van sirkel na middelpnt van koord] In ΔABC & ΔDOC i) $\hat{A} = \hat{D}_2$ [both equal to 90°]/[beide gelyk aan 90°] ii) $\hat{C} = \hat{C}$ [common/gemeen] $\therefore \Delta ABC \parallel \Delta DOC$ [AAA/HHH]	✓ S/R ✓ S/R ✓ S ✓ $\Delta^e \parallel \parallel / R$	(4)
11.2	$\frac{OC}{BC} = \frac{DC}{AC}$ $\Delta ABC \parallel \Delta DOC$ $OC = \frac{DC \cdot BC}{AC}$	✓ S/R	(1)
11.3	$AC^2 = BC^2 - AB^2$ [Pythagoras] $= 30^2 - 18^2$ $= 576$ $AC = 24 \text{ cm}$ $OC = \frac{DC \cdot BC}{AC}$ $= \frac{15 \times 30}{24} = 18,8 \text{ cm}$	✓ subst in eqn / R verv. in verg./R ✓ $AC = 24$ ✓ subst / vervang. ✓ answer / antwoord	(4)
			[9]

TOTAL / TOTAAL: 150