# MEMO 2023-12ƎD-00010(2) Geometric series

(1) Calculate the first three terms of the GS with $S_\infty = 54$ and $T_2 = 12$.

$$S_\infty = \frac{a}{1-r} \qquad T_n = a.r^{n-1}$$

$$\therefore \quad 54 = \frac{a}{1-r} \qquad \therefore \quad T_2 = a.r^{2-1}$$

$$\therefore \quad 54(1-r) = a \qquad \therefore \quad 12 = a.r^1$$

$$\therefore \quad 54 - 54r = \frac{12}{r} \qquad \therefore \quad \frac{12}{r} = a$$

$$\therefore \quad 54r - 54r^2 = 12$$

$$\therefore \quad 0 = 54r^2 - 54r + 12$$

$$\therefore \quad 0 = 9r^2 - 9r + 2 \qquad [\div 6]$$

$$\therefore \quad 0 = (3r-2)(3r-1)$$

$$\therefore \quad r = \frac{2}{3} \qquad \text{or} \qquad r = \frac{1}{3}$$

$$\text{but} \quad a = \frac{12}{r} \qquad a = \frac{12}{r}$$

$$\therefore \quad a = 12 \div \frac{2}{3} \qquad a = 12 \div \frac{1}{3}$$

$$\therefore \quad a = 12 \times \frac{3}{2} \qquad a = 12 \times \frac{3}{1}$$

$$\therefore \quad a = 18 \qquad a = 36$$

**∴ Sequence: 18 ; 12 ; 8** **∴ Sequence: 36 ; 12 ; 4**

(2) Consider the GS: $(k+2) + (k+2)^2 + (k+2)^3 + \ldots\ldots\ldots$

(a) For which value(s) of $k$ will the series converge?

**Convergent if:** $-1 < r < 1 \; ; r \neq 0$

$$r = (k+2)$$

$$\therefore \quad -1 < k+2 < 1 \quad ; \quad k+2 \neq 0$$

$$\therefore \quad -1-2 < k < 1-2 \quad ; \quad k \neq -2$$

$$\therefore \quad -3 < k < -1 \quad ; \quad k \neq -2$$

(b) Calculate, in terms of $k$, the sum to infinity of the series if it is convergent.

$a = k+2$ $\qquad S_\infty = \frac{a}{1-r}$

$r = (k+2)$ $\qquad \therefore \quad S_\infty = \frac{k+2}{1-(k+2)}$

$$\therefore \quad S_\infty = \frac{k+2}{1-k-2}$$

$$\therefore \quad S_\infty = \frac{k+2}{-k-1}$$

(3) Calculate:

(a) $2 - 4 + 8 - 16 + \ldots\ldots + 512$ **GS**

$\mathbf{a = 2}$ $\quad\quad$ $\mathbf{T_n = a.r^{n-1}}$

$\mathbf{r = -2}$ $\quad\therefore\quad$ $\mathbf{512 = 2 \times (-2)^{n-1}}$

$\mathbf{n = ?}$ $\quad\therefore\quad$ $\mathbf{\frac{512}{2} = (-2)^{n-1}}$

$\mathbf{T_n = 512}$ $\quad\therefore\quad$ $\mathbf{256 = (-2)^{n-1}}$

$\therefore\quad$ $\mathbf{(-2)^8 = (-2)^{n-1}}$

$\therefore\quad$ $\mathbf{n - 1 = 8}$ $\quad\rightarrow\quad$ **EB ⇔ EE**

$\therefore\quad$ $\mathbf{n = 9}$

$\mathbf{a = 2}$ $\quad\quad$ $\mathbf{S_n = \frac{a(r^n - 1)}{r - 1}}$

$\mathbf{r = -2}$ $\quad\therefore\quad$ $\mathbf{S_9 = \frac{2((-2)^9 - 1)}{-2 - 1}}$

$\mathbf{n = 9}$ $\quad\therefore\quad$ $\mathbf{S_9 = \frac{2((-2)^9 - 1)}{-3}}$

$\mathbf{S_n = ?}$ $\quad\therefore\quad$ $\mathbf{S_9 \approx 342}$

(b) $-38 - 35{,}5 - 33 - 30{,}5 - \ldots\ldots - 10{,}5$ **AS**

$\mathbf{a = -38}$ $\quad\quad$ $\mathbf{T_n = a + (n-1)d}$

$\mathbf{d = 2,5}$ $\quad\therefore\quad$ $\mathbf{-10,5 = -38 + (n-1)(2,5)}$

$\mathbf{n = ?}$ $\quad\therefore\quad$ $\mathbf{-10,5 + 38 = 2,5n - 2,5}$

$\mathbf{T_n = -10,5}$ $\quad\therefore\quad$ $\mathbf{27,5 + 2,5 = 2,5n}$

$\therefore\quad$ $\mathbf{30 = 2,5n}$

$\therefore\quad$ $\mathbf{n = \frac{30}{2,5}}$

$\therefore\quad$ $\mathbf{n = 12}$

$\mathbf{a = -38}$ $\quad\quad$ $\mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$

$\mathbf{d = 2,5}$ $\quad\therefore\quad$ $\mathbf{S_{12} = \frac{12}{2}[2 \times (-38) + (12-1)(2,5)]}$

$\mathbf{n = 12}$ $\quad\therefore\quad$ $\mathbf{S_{12} = 6[-76 + 11 \times 2,5]}$

$\mathbf{S_n = ?}$ $\quad\therefore\quad$ $\mathbf{S_{12} = -291}$

(c) $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots\ldots$ to seven term **GS**

$a = \frac{2}{3}$ $\qquad S_n = \frac{a(r^n - 1)}{r - 1}$

$r = \frac{1}{3}$ $\qquad \therefore \quad S_7 = \frac{\frac{2}{3}\left(\left(\frac{1}{3}\right)^7 - 1\right)}{\frac{1}{3} - 1}$

$n = 7$ $\qquad \therefore \quad S_7 = \frac{\frac{2}{3}\left(\left(\frac{1}{3}\right)^7 - 1\right)}{-\frac{2}{3}}$

$S_n = ?$ $\qquad \therefore \quad S_7 = \frac{2\,186}{2\,187}$

(d) $\sum_{n=5}^{12} \left(\frac{1}{2} \times 3^n\right)$

**Sequence:** $\frac{1}{2} \times 3^5 + \frac{1}{2} \times 3^6 + \frac{1}{2} \times 3^7 + \cdots + \frac{1}{2} \times 3^{12} = \frac{243}{2} + \frac{729}{2} + \frac{2\,187}{2} +$ ... **8 terms**

$a = \frac{243}{2}$ $\qquad S_n = \frac{a(r^n - 1)}{r - 1}$

$r = 3$ $\qquad \therefore \quad S_8 = \frac{\frac{243}{2}((3)^8 - 1)}{3 - 1}$

$n = 8$ $\qquad \therefore \quad S_8 = \frac{\frac{243}{2}((3)^8 - 1)}{2}$

$S_n = ?$ $\qquad \therefore \quad S_8 = 398\,520$

(4) The third term of a GS is 3 and the seventh term is $15\frac{3}{16}$. Calculate:

(a) the constant ratio of the sequence.

$T_3 = 3$ $\qquad T_7 = 15\frac{3}{16}$

$\therefore \quad T_3 = a \cdot r^{3-1}$ $\qquad \therefore \quad T_7 = a \cdot r^{7-1}$

$\therefore \quad 3 = a \cdot r^2 \ \ldots (1)$ $\qquad \therefore \quad 15\frac{3}{16} = a \cdot r^6 \ \ldots (2)$

$\therefore \quad \frac{\frac{243}{16}}{3} = \frac{a \cdot r^6}{a \cdot r^2}$ $\qquad (2) \div (1)$

$\therefore \quad \frac{81}{16} = \frac{\cancel{a} \cdot r^6}{\cancel{a} \cdot r^2}$

$\therefore \quad \frac{81}{16} = r^{6-2}$

$\therefore \quad \left(\frac{3}{2}\right)^4 = r^4$ $\qquad$ **but EB ⇔ EE**

$\therefore \quad r = \pm\frac{3}{2}$ $\qquad \rightarrow \ \pm$ **because exponent is even**

(b) the first term of the sequence.

**Substitute r in (1) or (2):**

$\therefore \quad a.r^2 = 3 \quad \ldots\ldots (1)$

$\therefore \quad a.\left(\pm\frac{3}{2}\right)^2 = 3$

$\therefore \quad a.\frac{9}{4} = 3$

$\therefore \quad a = 3 \times \frac{4}{9}$

$\therefore \quad a = \frac{4}{3}$

**or**

$15\frac{3}{16} = a.r^6 \quad \ldots (2)$

$a.\left(\pm\frac{3}{2}\right)^6 = \frac{243}{16}$

$a.\frac{729}{64} = \frac{243}{16}$

$\therefore \quad a = \frac{243}{16} \times \frac{64}{729}$

$\therefore \quad a = \frac{4}{3}$

(c) which term is equal to $76\frac{227}{256}$.

$a = \frac{4}{3}$

$r = \pm\frac{3}{2}$

$n = ?$

$T_n = 76\frac{227}{256}$

$T_n = a.r^{n-1}$

$\therefore \quad 76\frac{227}{256} = \frac{4}{3} \times \left(\pm\frac{3}{2}\right)^{n-1}$

$\therefore \quad \frac{19\,683}{256} = \frac{4}{3} \times \left(\pm\frac{3}{2}\right)^{n-1}$

$\therefore \quad \frac{19\,683}{256} \times \frac{3}{4} = \left(\pm\frac{3}{2}\right)^{n-1}$

$\therefore \quad \frac{59\,049}{1\,024} = \left(\pm\frac{3}{2}\right)^{n-1}$

$\therefore \quad \left(\pm\frac{3}{2}\right)^{10} = \left(\pm\frac{3}{2}\right)^{n-1}$ but EB $\Leftrightarrow$ EE

$\therefore \quad 10 = n - 1$

$\therefore \quad n = 11$

(5) Calculate $n$ if $3 + 12 + 48 + 192 + \ldots\ldots\ldots +$ to $n$ terms $= 1\,048\,575$

$a = 3$

$r = 4$

$n = ?$

$S_n = 1\,048\,575$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore \quad 1\,048\,575 = \frac{3(4^n - 1)}{4 - 1}$

$\therefore \quad 1\,048\,575 = \frac{3(4^n - 1)}{3}$

$\therefore \quad 1\,048\,575 = \frac{3(4^n - 1)}{3}$

$\therefore \quad 1\,048\,575 = 4^n - 1$

$\therefore \quad 1\,048\,576 = 4^n$

$\therefore \quad 4^{10} = 4^n$ but EB $\Leftrightarrow$ EE

$\therefore \quad n = 10$

(6) Consider the series: $384 + 192 + 96 + 48 + \ldots\ldots$

(a) Write the series in sigma notation.

$a = 384$ $\qquad$ $T_n = a.r^{n-1}$

$r = \frac{1}{2}$ $\qquad$ $\therefore\ T_n = 384.\left(\frac{1}{2}\right)^{n-1}$

$n = ?$ $\qquad$ $\therefore\ T_n = 384.(2^{-1})^{n-1}$

$T_n = ?$ $\qquad$ $\therefore\ T_n = 384.(2)^{1-n}$

$$\therefore \sum_{n=1}^{k} (384 \times 2^{1-n})$$

(b) Determine $S_\infty$, if possible.

$$S_\infty = \frac{a}{1-r}$$

$$\therefore\ S_\infty = \frac{384}{1-\frac{1}{2}}$$

$$\therefore\ S_\infty = 768$$

(c) Calculate $T_{12}$.

$$T_n = 384.(2)^{1-n} \quad \rightarrow \quad \text{sien (a)}$$

$$\therefore\ T_{12} = 384.(2)^{1-12}$$

$$= 384.(2)^{-11}$$

$$\therefore\ T_{12} = \frac{3}{16}$$

(d) Determine how many terms will have a sum of 762.

$a = 384$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r-1}$

$r = \frac{1}{2}$ $\qquad$ $\therefore\ 762 = \frac{384\left(\left(\frac{1}{2}\right)^n - 1\right)}{\frac{1}{2} - 1}$

$n = ?$ $\qquad$ $\therefore\ 762 = \frac{384\left(\left(\frac{1}{2}\right)^n - 1\right)}{-\frac{1}{2}}$

$S_n = 762$ $\qquad$ $\therefore\ 762 = -768\left(\left(\frac{1}{2}\right)^n - 1\right)$

$$\therefore\ \frac{762}{-768} = \left(\frac{1}{2}\right)^n - 1$$

$$\therefore\ -\frac{127}{128} = \left(\frac{1}{2}\right)^n - 1$$

$$\therefore\ 1 - \frac{127}{128} = \left(\frac{1}{2}\right)^n$$

$$\therefore\ \frac{1}{128} = \left(\frac{1}{2}\right)^n$$

$$\therefore\ \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^n \quad \text{but} \quad EB \Leftrightarrow EE$$

$$\therefore\ n = 7$$