MEMO 2015-9ƎC-00003(2)

(1) (i) Write the next four numbers in each sequence.
(ii) Describe the pattern in words.
(iii) Determine the 100^{th} term of each sequence.

(a) 84 ; 184 ; 284 ; 384 ; **484 ; 584 ; 684 ; 784**

(ii) **Add 100.** → $+100$

(iii) $T_n = a + (n-1)d$

$\therefore T_{100} = 84 + (100-1)(100)$

$\therefore T_{100} = 84 + 99 \times 100 = 84 + 9\,900$

$\therefore T_{100} = 9\,984$

(b) 1 ; 8 ; 27 ; 64 ; **125 ; 216 ; 343 ; 512**

(ii) **Cubes.** → n^3

(iii) $T_n = n^3$

$\therefore T_{100} = 100^3$

$\therefore T_{100} = 1\,000\,000$

(c) 4,75 ; 4,81 ; 4,87 ; 4,93 ; **4,99 ; 5,05 ; 5,11 ; 5,17**

(ii) **Add 0,06.** → $+0,06$

(iii) $T_n = a + (n-1)d$

$\therefore T_{100} = 4,75 + (100-1)(0,06)$

$\therefore T_{100} = 4,75 + 99 \times 0,06 = 4,75 + 5,94$

$\therefore T_{100} = 10,69$

(d) $\frac{3}{64}$; $\frac{3}{32}$; $\frac{3}{16}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{3}{2}$; 3 ; 6

(ii) **Multiply by 2.** $\rightarrow$ $\times 2$

(iii) $T_n = \frac{3}{64} \times 2^{n-1}$ $\rightarrow$ $T_1 = \frac{3}{64} \times 2^{1-1} = \frac{3}{64} \times 2^0 = \frac{3}{64} \times 1 = \frac{3}{64}$

$\therefore$ $T_{100} = \frac{3}{2^6} \times 2^{100-1}$

$\therefore$ $T_{100} = \frac{3}{2^6} \times 2^{99}$

$\therefore$ $T_{100} = 3 \times 2^{99-6}$

$\therefore$ $T_{100} = 3 \times 2^{93}$

(e) -133 ; -137 ; -141 ; -145 ; -149 ; -153 ; -157 ; -161

(ii) **Subtract 4.** $\rightarrow$ -4

(iii) $T_n = a + (n-1)d$

$\therefore$ $T_{100} = -133 + (100-1)(-4)$

$\therefore$ $T_{100} = -133 + 99 \times -4 = -133 - 396$

$\therefore$ $T_{100} = -529$

(2) Complete the following tables and write the relation between x and y:

(a)

x	1	2	3	6	8	13	16	21	108	118
y	18	25	32	53	67	102	123	158	767	837

+1 +1 +3 +5 +(609 ÷ 7)

+7 +7 +21 = 3 × 7 +35 = 5 × 7 609

$T_0 = 18 - 7 = 11$ or $T_n = a + (n-1)d$

$\therefore$ $y = 7x + 11$ $\therefore$ $y = 18 + (x-1)(7)$

$\therefore$ $y = 18 + 7x - 7$

$\therefore$ $y = 7x + 11$

(b)

x	5	6	7	9	12	20	26	33	39	111
y	−14	−16	−18	−22	−28	−44	−56	−70	−82	−226

+1, +1, +2 ; −14 ÷ −2 = +7 ; −2, −2, −4 = 2 × −2 ; −14 ; 5 − 5 = 0

$T_0 = -14 + 5 \times 2$

$= -14 + 10 = -4$

$\therefore \quad y = -2x - 4$

or

T_5

$T_1 = -14 + 2 + 2 + 2 + 2 = -6$

$T_n = a + (n-1)d$

$\therefore \quad y = -6 + (x-1)(-2)$

$\therefore \quad y = -6 - 2x + 2$

$\therefore \quad y = -2x - 4$

(c)

x	1	3	5	7	9	11	19	30	35	52
y	5	15	25	35	45	55	95	150	175	260

$y = x \times 5 \quad \rightarrow \quad x = \frac{y}{5}$

$\therefore \quad y = 5x$

(3) The third, fourth and fifth terms of a sequence are respectively 57 ; 54 and 51.

(a) Write the first and second terms of the sequence.
(b) Determine the general term of the sequence.
(c) Determine the difference between T_{80} and T_{81}.
(d) What will the first negative term in the sequence be?

(a) $T_1 = 63$, $T_2 = 60$, $T_3 = 57$, $T_4 = 54$, $T_5 = 51$; +3, +3, −3, −3

(b) $T_n = a + (n-1)d$

$\therefore \quad T_n = 63 + (n-1)(-3)$

$\therefore \quad T_n = 63 - 3n + 3$

$\therefore \quad T_n = -3n + 66$

(c) $T_{81} - T_{80} = T_4 - T_3 = T_2 - T_1 = -3$ → Constant difference

(d) First negative term is if $T_n < 0$

$\therefore \quad -3n + 66 < 0$

$\therefore \quad 66 < 3n$

$\therefore \quad 3n > 66$

$\therefore \quad n > \frac{66}{3}$

$\therefore \quad n > 22$

$\therefore$ The first negative term will be T_{23}.